

Formal Home Health Care, Informal Care, and Family Decision Making*

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Abstract

We use the 1993 wave of the AHEAD data set to estimate a game-theoretic model of families' decisions concerning the provision of informal and formal care for elderly individuals. The outcome is a Nash equilibrium where each family member jointly determines her consumption, transfers for formal care, and allocation of time to informal care, market work, and leisure. We use the estimates to decompose the effects of adult children's opportunity costs, quality of care, and caregiving burden on their propensities to provide informal care. We also simulate the effects of a broad range of policies of current interest.

Keywords: Long-term Care, Home Health Care, Informal Care, Empirical Game Theory

JEL Classifications: C51, C72, J14

Shortened Title: Families' Elder Care Decisions

1 Introduction

Increased life expectancies in recent decades have contributed to the aging of the population. Between 1980 and 2000, for example, the elderly population, defined as individuals aged 65 years and older, increased by 37 percent. Demographers predict that the elderly population will reach 71 million, or 20 percent of the total population, by 2030. As of 2004, the oldest old population, those 85 years and older, was growing three times faster than the general population (U.S. Census Bureau 2004). Although disability rates among the elderly decreased between 1982 and 1999 (Manton and Gu 2001), the number of disabled

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elderly individuals has remained approximately constant at 5.5 million because of population aging and the increased level of disability among those receiving long-term care (Spector, et al. 2001).

Population aging has coincided with dramatic changes in care arrangements for the elderly. Informal care (i.e., unpaid care) has become less common, while formal home health care (i.e., paid care) and institutional care have become more widespread (Boersch-Supan, et al. 1988, Wolf and Soldo 1988). For example, about 25 percent of the oldest old lived in institutions in 1990 compared to 7 percent in 1940 (Kotlikoff and Morris 1990). Although formal home health care was relatively uncommon until recent decades, 1.4 million individuals received this form of care in 2000. Between 1989 and 1999, the number of informal caregivers rose only 6 percent, whereas the elderly population increased 13 percent (Mack and Thompson 2004). Despite the trends toward institutional and formal home health care, adult children and spouses continue to enable elderly individuals to remain in the community; in fact, most elderly who remain in the community do so with the assistance of familial and social networks (e.g., Matthews and Rosner 1988).

Elder care arrangements have profound economic, social, and psychological implications. The high cost of institutional care often exhausts the resources of nursing home residents. As a consequence, many elderly individuals and their families rely on Medicaid to cover their long-term care expenses. In addition to the financial burden borne by families and by society, institutional care typically involves social and psychological costs for elderly individuals (Macken 1986). Although less expensive than institutional care, home health care consumes an increasing share of health care expenditures (National Center for Health Statistics 1996; U.S. Department of Health and Human Services 2000). Care provided by family members typically does not impose explicit financial costs, but the opportunity costs in terms of foregone earnings or nonmarket time can be substantial. Also, the provision of informal care can be psychologically burdensome for caregivers.

In light of population aging, the changing patterns of elder care, and the profound implications of care arrangements for the recipients of care, their families, and society, the development of appropriate public policies requires an understanding of families' elder care decisions. We focus on the provision of informal and formal home health care for the noninstitutionalized elderly. Specifically, we construct a game-theoretic model of family decision making where each family member makes decisions concerning the provision of informal and formal care as part of a broader utility maximization problem. We use the 1993 wave of the AHEAD data to estimate our game-theoretic model. Results of this model provide insight concerning the role of demographic characteristics and public policies in families' care decisions and the welfare of family members. We use the results to simulate the effects of subsidizing informal and formal care and relaxing the requirements for Medicaid qualification. The model is an early step in developing and estimating structural models of family decision making and long-term care decisions.

2 Literature Review

Although predominantly empirical, the literature on caregiving for the elderly offers several theoretical models. These models vary along several dimensions: whether family members share common preferences, which family members participate in the decision-making process, which types of care arrangements are considered, and whether other decisions are determined jointly with parental care decisions.

Some papers in the elder care literature assume that a single household utility function is appropriate in the context of elderly parents and their adult children. For example, Hoerger, Picone, and Sloan's (1996) (HPS) model involves a family utility function and budget constraint.¹ Some of the other models,² including the one presented in this study, are game-theoretic and thus involve separate utility functions for each family member.

Several of the existing theoretical models involve only one child in the decision-making process.³ This assumption considerably simplifies modeling and estimation but obscures the dynamics within the younger generation. In practice, more than one adult child in a family may participate in the family's care decision, and adult siblings may disagree regarding the best source of care for an elderly parent. The potential disagreement among adult siblings and between adult children and elderly parents motivates the development of a game-theoretic framework where the players include the parent, spouse, and all of her⁴ children. Moreover the burden associated with caregiving may generate strategic interaction among family members. For example, an adult child's provision of informal care for her father may depend on the amount of informal care provided by her siblings and by her mother. Although altruistic toward her father, the adult child may have incentive to free ride on her siblings' or her mother's informal care. Thus, her provision of informal care may depend negatively on the amount of care provided by other family members. Alternatively, in the spirit of Bernheim, Schleifer, and Summers (1985), a bequest motive could induce siblings to compete with one another for a greater share of the inheritance. Thus, an adult child's provision of informal care could depend positively on the amount of care provided by a sibling. Similarly, siblings may have incentive to free ride on one another with respect to financial transfers for formal home health care. The possibility of such strategic play suggests that a non-cooperative model may be appropriate in the context of families' caregiving decisions for the elderly.

As part of an effort to develop more realistic models of family decision mak-

¹In Kotlikoff and Morris (1990), parent and child solve separate maximization problems if they live separately but maximize a weighted average of their individual utility functions subject to their pooled budget constraint if they live together.

²See Pezzin and Schone (1997, 1999), Sloan, Picone, and Hoerger (1997), Hiedemann and Stern (1999), Checkovich and Stern (2002), and Engers and Stern (2002).

³Pezzin and Schone (1997, 1999) and Sloan, Picone, and Hoerger (1997) present models that apply to families of any size, but only one child plays a role in the family's care decision.

⁴Throughout the paper, we use female pronouns as the generic pronouns. This does not mean that only mothers need care or that only daughters provide care.

ing, Hiedemann and Stern (1999) (HS), Checkovich and Stern (2002) (CS), Engers and Stern (2002) (ES), and the current study present game-theoretic models that accommodate a variable number of children and the possibility that all children play a role in care decisions. Whereas HS and ES develop and estimate stylized games that cannot be identified from one another given the available data (ES), the current paper considers a much more intuitive game and equilibrium. Here each agent maximizes a relatively standard utility function in the context of a Nash equilibrium. The current paper also differs from previous work with respect to the scope of care decisions modeled. HS and ES model the decision to provide informal care, while CS model the quantity of informal care provided. Here we consider both of these choices – whether and how much informal care to provide – in a broader utility maximization framework. In the current model, family members make informal care decisions jointly with decisions concerning financial contributions for home health care, consumption, market work, and leisure.

Given the variety of care arrangements and the connection between care arrangements and living arrangements, one model cannot capture all possible aspects of a family’s parental care and living arrangements. While Pezzin and Schone (1997), Sloan, Picone, and Hoerger (1997) (SPH), HS, CS, and ES focus on care arrangements, HPS, and Pezzin and Schone (1999)(PS) model both care and living arrangements.⁵ We present a model in which each family member decides how much informal and formal home health care to provide for elderly parents, taking living arrangements as given. This study is most closely related to those of SPH, PS, and CS. PS jointly model living arrangements with the provision of care by the child (in this case, a daughter). SPH present a model in which the choice variables are not the type of care or living arrangement but hours of formal care and informal care provided by the child. CS model each child’s provision of informal care. Finally, the provision of care by adult children may be determined simultaneously with labor force behavior. As in our study, Ettner (1996) and Pezzin and Schone (1997, 1999) model labor force participation of adult children jointly with care and/or living arrangements.⁶

The econometric models in the elder care literature are as varied as the theoretical models. Most papers present results based on nonstructural models.⁷ But several recent papers present results based on structural models.⁸ With

⁵In a related literature, Kotlikoff and Morris (1990) focus on living arrangements including residence in a nursing home.

⁶The long term care literature addresses other factors that may play a role in the family’s care decisions. For instance, inter- or intragenerational transfers may be made as part of a family’s long-term care decision. This possibility may be captured by assuming that the family pools its income (e.g., HPS) or by explicitly modeling side payments among family members. PS model intergenerational cash transfers jointly with caregiving, intergenerational household formation, and labor force behavior. In one of the models in ES, family members choose the long-term care alternative that maximizes their joint payoff and make any necessary side payments among themselves.

⁷See Wolf and Soldo (1988), Lee, Dwyer, and Coward (1990), Cutler and Sheiner (1993), Ettner (1996), HPS, Boaz and Hu (1997), Diwan, Berger, and Manns (1997), Norgard and Rodgers (1997), SPH, White-Means (1997), and Couch, Daly, and Wolf (1999).

⁸See Pezzin and Schone (1997, 1999), HS, CS, and ES.

the exception of CS and this paper, existing studies focus on the role of a single child in each family as the primary caregiver and ignore the possibility of other children serving as sources of assistance.⁹ However, data from the 1984 National Long-term Care Survey indicate that shared caregiving is an important phenomenon, especially in large families. CS show, for example, that over 4 percent of families with two children, almost 10 percent of families with three children, and about 16 percent of families with four children contain multiple caregivers. Among families where at least one child provides care, the probability that children share caregiving is almost 13 percent in families with two children, over 25 percent in families with three children, and almost 35 percent in families with four children. Even if each family relies on a single caregiver, one cannot ignore the other children in the family. Children attempt to influence both the amount and the method of caregiving provided by their siblings. Not only are there possibilities for intersibling conflict as a result of parental care provision, but a large majority of distant children report emotional support received from siblings regarding the situation of their disabled parent (Schoonover, Brody, Hoffman, and Kleban 1988).

3 Medicaid Financing Rules

For many households, provision of formal and informal care depends on available public assistance, most notably Medicaid. Medicaid is a joint federal/state, means-tested entitlement program that finances medical assistance to individuals with low income. Federal contributions to each state vary according to a matching rule that depends on which medical services are financed by the state. Medicaid is estimated to have served 31.4 million individuals in fiscal year (FY) 1992, at a combined cost of \$118.8 billion, about 15 percent of total national health spending (Congressional Research Service 1993, p. 1).

Eligibility for Medicaid is linked to actual or potential receipt of cash assistance under the Supplemental Security Income (SSI) program or the former Aid to Families with Dependent Children (AFDC) program. Elderly individuals are eligible for SSI payments if their monthly countable income (income less \$20) and countable resources fall below standards set by federal law. In 1993, the year of our sample, the SSI income limit was \$434 per month for individuals and \$652 per month for couples. The 1993 SSI resource limits were \$2000 for individuals and \$3000 for couples.

In designing their Medicaid programs, states must adhere to federal guidelines. Even so, variation among state programs is considerable. Byrne, Goree, Hiedemann, and Stern (2003) provide information on the variation in rules across states. Eligibility in each state depends on the state's policies with regard to three main groups: individuals classified as categorically or medically needy and individuals residing in medical care institutions or needing home and

⁹See Frankfather, Smith, and Caro (1981), Johnson and Catalano (1981), Cantor (1983), Johnson (1983), Stoller and Earl (1983), Horowitz (1985), Barber (1989), Miller and Montgomery (1990), Stern (1994, 1995, 1996), Pezzin and Schone (1997, 1999), HS, and ES.

community-based care.

When determining Medicaid categorical eligibility, states have the option of supplementing the federal income standard. The State Supplement Payments (SSP) are made solely with state funds. The combined federal SSI and state SSP benefit becomes the effective income eligibility standard. Alternatively, states may use more restrictive eligibility standards than those for SSI if they were using those standards prior to the implementation of SSI.

As mentioned above, Medicaid also allows states to cover individuals who are not poor by the relevant income standard but who need assistance with medical expenses. To qualify for medically needy coverage, individuals must first deplete their resources to the state's standard and must have high medical expenses relative to the income level required by the state. States are permitted by federal law to establish a special income standard for individuals who are residents of nursing facilities or other institutions. The special income limit may not exceed 300 percent of the maximum SSI benefit. In states without a medically needy program, this "300 percent rule" is an alternative way of providing coverage to individuals with incomes above the state's limit.

Finally, under the Section 1915c waiver program, states have the option of covering individuals needing home and community-based care services if these individuals would otherwise require institutional care covered by Medicaid. States use waiver programs to provide services to a diverse long-term care population, including the elderly. Spending for 1915c waiver services has grown dramatically since the enactment of the law in 1981. Federal and state spending increased from \$3.8 million in FY 1982 to \$1.7 billion in FY 1991 (Congressional Research Service 1993, p. 400). Equivalently, about 13 percent of Medicaid long-term care spending covered home and community based care in 1991.

4 Theoretical Model

4.1 The Model

We develop and estimate a game-theoretic model of the provision of formal home health care and informal care for elderly individuals. In our model, family members from two generations participate in the decision making process. The decision makers include an elderly individual or couple and her/their children and children-in-law. Each family member has the opportunity to make financial contributions for formal home health care and to spend time providing informal care. Thus, the model accommodates the possibility of multiple caregivers.

Family members make caregiving decisions as part of a broader utility maximization framework. The younger generation allocates time to market work, informal care, and leisure and allocates money to consumption and formal care. The older generation no longer participates in the labor market and thus faces one fewer choice variable. In addition to consumption and leisure, utility depends on time spent providing informal care and on the health quality of the

elderly individual(s). In turn, an elderly individual's health quality is a function of both informal and formal care as well as demographic characteristics. Preferences concerning the provision of care may vary across generations and among siblings, but married couples are assumed to share a single set of preferences.

The outcome is a Nash equilibrium where each family member maximizes utility subject to budget and time constraints, taking as given the other family members' behavior. Thus, each individual's or couple's provision of formal and informal care depends on the care provided by the other family members.

The model (and data) allow us to distinguish among three important sources of variation in care provision across families. First, some family members may find caregiving burdensome. To the extent that caregiving is burdensome, family members may have incentive to free ride on one another in the provision of care. Second, some family members may provide higher quality care than others. Third, opportunity costs in the form of foregone earnings may vary across family members, resulting in different choices of care provision.

More technically, consider a family¹⁰ with I adult children and one or two elderly parents. The family includes between $I+1$ and $2(I+1)$ adults depending on the marital status of the parent and each child. As mentioned above, we assume that married couples act as a single player; thus, there are $I+1$ players indexed by $i = 0, 1, 2, \dots, I$. When indexing married players, we use m and p for maternal and paternal and c and s for child and spouse. The term a_{ik} ($k = m, p$ for parents, and $k = c, s$ for children) takes the value 1 if the family includes the individual in question and 0 otherwise. For example, $a_{1s} = 1$ if child 1 is married, and $a_{1s} = 0$ if the child is not married. As discussed earlier, each player makes decisions about consumption X_i , contributions for formal home health care (measured in time units) H_i , leisure L_{ik} , and time spent caring for the mother t_{mik} and father t_{pik} , where $k = c, s$ for children and their spouses. The children also determine their market work time, but the parents no longer participate in the labor market. For the parents, t_{p0m} is care provided for the father by the mother, and t_{m0p} is care provided for the mother by the father. We assume at least one of t_{m0p} and t_{p0m} is zero, and, if there is only one parent, both are zero. Finally, parents do not care for themselves; hence t_{m0m} and t_{p0p} are both zero. Market work time is $1 - L_{ik} - \sum_{j \in m, p} t_{jik}$ for the children and their spouses and zero for parents.

¹⁰For now, we suppress a family index n that will appear in the Estimation Strategy section.

Health quality production functions,

$$\begin{aligned}
Q_m &= a_{0p}\alpha_{m0p} (t_{m0p} + \gamma t_{m0p}^2) + \sum_{i=1}^I \sum_{k \in c,s} a_{ik}\alpha_{mik} (t_{mik} + \gamma t_{mik}^2) \quad (1) \\
&+ \mu \sum_{i=0}^I H_i + Z_m \quad \text{and} \\
Q_p &= a_{0m}\alpha_{p0m} (t_{p0m} + \gamma t_{p0m}^2) + \sum_{i=1}^I \sum_{k \in c,s} a_{ik}\alpha_{pik} (t_{pik} + \gamma t_{pik}^2) \\
&+ \mu \sum_{i=0}^I H_i + Z_p,
\end{aligned}$$

determine the health quality of each parent where Z_j is the exponent of a linear combination of parent j 's characteristics. The parameters α_{jik} , γ , and μ measure the effects of care provided by family members (informal care) and paid care (formal care) on health quality.¹¹ The α_{jik} coefficients may depend on observed parent and child characteristics. The health quality terms, Q_m and Q_p , represent aggregate measures of true health (such as problems with ADLs) and accommodations made for health problems.¹² Informal care t may not influence true health per se but may help the parent deal with health problems thus impacting "health quality." Both of these effects are captured in equation (1) and cannot be identified separately given data constraints. Finally, informal care may simply make the parent happier.

The parents' utility function¹³ takes the form

$$\begin{aligned}
U_0 &= \beta_0 + \beta_{10} \sum_{j \in m,p} a_{0j} \ln Q_j + \beta_{20} \varepsilon_{X0} \ln X_0 + \sum_{k \in m,p} a_{0k} \beta_{30k} \varepsilon_{L0k} \ln L_{0k} \\
&+ \sum_{\substack{j,k \in m,p \\ j \neq k}} a_{0k} a_{0j} (\beta_{4j0k} + \varepsilon_{t0jk}) t_{j0k} + \varepsilon_{u0}. \quad (2)
\end{aligned}$$

¹¹To be clear about terminology, we use health quality to refer to Q and quality of care to refer to the impact of formal and informal care on health quality.

¹²We do not have direct measures of health quality (Q); rather we observe the output of health quality indirectly through its effect on utility.

¹³In the estimation section, we will have occasion to define the utility function of each parent. We define the utility of parent j as

$$U_{0j} = \beta_{0j} + \beta_{10} \ln Q_j + \beta_{20} \varepsilon_{X0} \zeta \ln X_0 + \beta_{30j} \varepsilon_{L0j} \ln L_{0j} + (\beta_{40jk} + \varepsilon_{t0jk}) t_{0jk} + \varepsilon_{u0j}$$

where $k = p$ if $j = m$ and $k = m$ if $j = p$ and $\zeta = .5$ if k is alive and $\zeta = 1$ if k is not alive.

Similarly, child i 's utility function (for $i > 0$) takes the form¹⁴

$$U_i = \beta_0 + \beta_{1i} \sum_{j \in m,p} a_{0j} \ln Q_j + \beta_{2i} \varepsilon_{Xi} \ln X_i + \sum_{k \in c,s} a_{ik} \beta_{3ik} \varepsilon_{Lik} \ln L_{ik} \quad (3)$$

$$+ \sum_{k \in c,s} \sum_{j \in m,p} a_{ik} a_{0j} (\beta_{4jik} + \varepsilon_{tjik}) t_{jik} + \varepsilon_{ui}.$$

The coefficients β_0 , β_{1i} , β_{2i} , β_{3ik} , and β_{4jik} for $i = 0, 1, 2, \dots, I$ may depend on observed child and parent characteristics, and the errors ε_{Xi} , ε_{Lik} , and ε_{tjik} are functions of unobserved (to the econometrician) child and parent characteristics. All variables, including errors, are common knowledge to all family members. Each family member's utility depends positively on the parents' health quality as well as the family member's consumption and leisure. Thus, $\beta_{1i} \geq 0$, $\beta_{2i} \geq 0$, $\beta_{3ik} \geq 0$, $\varepsilon_{Xi} \geq 0$, and $\varepsilon_{Lik} \geq 0$ for $i = 0, 1, 2, \dots, I$.

Note that happiness and health quality may differ. The structure of the model allows an elderly individual to experience a high quality of health while expressing unhappiness. For example, an elderly woman with high health quality may express unhappiness if her husband's health quality is poor or if she experiences burden taking care of him. In the case of an unmarried elderly individual, high health quality may coincide with unhappiness if the marginal utility of health quality is low or if consumption is low.

Each player maximizes U_i over its choices subject to budget and time constraints taking as given the decisions of the other family members. Children and their spouses face budget constraints of the form

$$\max [Y_i^*, Y_i^{**}] \geq p_{Xi} X_i + q H_i \quad (4)$$

where p_{Xi} is the price of the consumption good, q is the price of a unit of paid care assistance purchased in the parent's state of residence,

$$Y_i^* = \sum_{k \in c,s} a_{ik} w_{ik} \left(1 - L_{ik} - \sum_{j \in m,p} t_{jik} \right) \quad (5)$$

is labor income,

$$Y_i^{**} = Y_i + s Y_i^* \quad (6)$$

is income net of a hypothetical negative income tax ($0 < s < 1$), and w_{ik} is the market wage. Y_i is outside income including government welfare payments, and the time constraint is implied by the definition of market work time. We use the structure in equations (4), (5), and (6) because there are some children with

¹⁴The model in Bernheim, Schleifer, and Summers (1985) would imply that the utility child i receives from providing informal care depends directly on the amount of care provided by siblings. McGarry (1999) and CS reject the implication of Bernheim, Schleifer, and Summers (1985). Norton and van Houtven (2006) show that inter vivos transfers are positively correlated with provision of informal care. However, this, by itself, does not imply that children's informal care decisions should be correlated; it can mean that the parent is compensating the informal caregiver for her time.

no observed income in the data. The utility function in equation (3) implies that consumption is always positive, so we need to force children's income to be positive. We use the negative income tax structure implied by equation (6) as a crude approximation of reality. We estimate Y_i and s using CPS data and allow it to vary across states. The standard nonnegativity constraints also apply: $t_{jik} \geq 0$, $L_{ik} \geq 0$, $H_i \geq 0$, and $X_i \geq 0$ for $k = c, s$ and $i = 1, 2, \dots, I$.

For the parent, the budget constraint is

$$Y_0 \geq p_{X0}X_0 + qH_0 \quad (7)$$

if she is not eligible for Medicaid reimbursement of home health care expenses. If she is eligible, the budget constraint is

$$\Psi + q \min(\bar{H}, H_0) \geq p_{X0}X_0 + qH_0$$

where Ψ is the income limit and $q\bar{H}$ is the maximum reimbursable amount for home health care expenses. As discussed in Section 3, eligibility requirements and maximum reimbursable amounts vary across states. Since we know the parent's state of residence, we use the relevant policy variables in determining her budget constraint. This approach potentially allows us to be more precise (relative to studies using aggregate state data) about the effects of changes in Medicaid policy on families, since the impact may differ significantly by state.¹⁵

The parents' time constraints are

$$1 \geq L_{0k} + t_{j0k}, \quad j, k = m, p; \quad j \neq k$$

where L_{0k} is the leisure time of parent k . This implies that $t_{j0k} = 1 - L_{0k}$ for $j, k = m, p$ and $j \neq k$. The standard nonnegativity constraints apply here as well: $t_{j0k} \geq 0$, $L_{0k} \geq 0$, $H_0 \geq 0$, and $X_0 \geq 0$ for $k = m, p$.

4.2 Family Equilibrium and First Order Conditions

The outcome of the game is a Nash equilibrium. The errors are functions of characteristics unobservable by the econometrician. For each child, we can solve for X_i using equation (4) to obtain

$$X_i = \frac{\max[Y_i^*, Y_i^{**}] - qH_i}{p_{Xi}}. \quad (8)$$

For the parent, using equation (7), we obtain

$$X_0 = \frac{Y_0 - qH_0}{p_{X0}}.$$

¹⁵Thirteen percent of respondents report they have an insurance policy that covers long-term care or home care. These respondents are somewhat less likely to report receipt of ADL assistance in their homes, probably because the elderly with coverage enter institutional care at a lower level of need. We control for ADL problems in the model but do not include long-term care insurance because we do not have enough information in the data to identify the choice to purchase it.

The model accommodates the possibility that family members may not contribute financial resources H_i or time for caregiving t_{jik} . Thus, for each child, the set of first order conditions (FOCs) for H_i is

$$\frac{\partial U_i}{\partial H_i} \leq 0, H_i \geq 0, \frac{\partial U_i}{\partial H_i} H_i = 0$$

and the FOCs for t_{jik} and L_{ik} depend on H_i .

We can summarize the set of first order conditions for the children as

FOCs for Children							
Cases				FOCs			
L_{ik}	t_{jik}	Work	H_i	H_i	t_{jik}	L_{ik}	
Int	Int	Int	Int	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t1}(t_{jik})$	$\varepsilon_{L_{ik}} = T_{ik}^{L1}$	
Int	Int	Int	Cor	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t2}(t_{jik}, \varepsilon_{Xi})$	$\varepsilon_{L_{ik}} = T_{ik}^{L2}(\varepsilon_{Xi})$	
Int	Int	Cor	Int	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t3}(t_{jik}, \varepsilon_{L_{ik}})$	$\varepsilon_{L_{ik}} \geq T_{ik}^{L1}$	
Int	Int	Cor	Cor	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t3}(t_{jik}, \varepsilon_{L_{ik}})$	$\varepsilon_{L_{ik}} \geq T_{ik}^{L2}(\varepsilon_{Xi})$	
Int	Cor	Int	Int	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t1}(0)$	$\varepsilon_{L_{ik}} = T_{ik}^{L1}$	
Int	Cor	Int	Cor	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t2}(0, \varepsilon_{Xi})$	$\varepsilon_{L_{ik}} = T_{ik}^{L2}(\varepsilon_{Xi})$	
Cor	Cor	Cor	Int	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t3}(0, \varepsilon_{L_{ik}})$	$\varepsilon_{L_{ik}} \geq T_{ik}^{L1}$	
Cor	Cor	Cor	Cor	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t3}(0, \varepsilon_{L_{ik}})$	$\varepsilon_{L_{ik}} \geq T_{ik}^{L2}(\varepsilon_{Xi})$	

where ‘‘Int’’ denotes an interior solution and ‘‘Cor’’ denotes a corner solution with

$$\begin{aligned} T_i^H &= \frac{\beta_{1i} \mu p_{Xi} X_i \bar{Q}}{\beta_{2i} q} \\ T_{ijk}^{t1}(t_{jik}) &= \frac{\beta_{1i} \mu s_i^* w_{ik} \bar{Q}}{q} - \beta_{1i} \tilde{\alpha}_{jik} - \beta_{4jik} \\ T_{ijk}^{t2}(t_{jik}, \varepsilon_{Xi}) &= \frac{\varepsilon_{Xi} \beta_{2i} s_i^* w_{ik}}{p_{Xi} X_i} - \beta_{1i} \tilde{\alpha}_{jik} - \beta_{4jik} \\ T_{ijk}^{t3}(t_{jik}, \varepsilon_{L_{ik}}) &= \frac{\varepsilon_{L_{ik}} \beta_{3ik}}{L_{ik}} - \beta_{1i} \tilde{\alpha}_{jik} - \beta_{4jik} \\ T_{ik}^{L1} &= \frac{\beta_{1i} \mu L_{ik} s_i^* w_{ik} \bar{Q}}{\beta_{3ik} q} \\ T_{ik}^{L2}(\varepsilon_{Xi}) &= \frac{\varepsilon_{Xi} \beta_{2i} L_{ik} s_i^* w_{ik}}{\beta_{3ik} p_{Xi} X_i} \end{aligned}$$

where

$$\begin{aligned} \tilde{\alpha}_{jik} &= \frac{\alpha_{jik} (1 + 2\gamma t_{jik})}{Q_j}, \\ \bar{Q} &= \sum_{j=m,p} \frac{a_{0j}}{Q_j} \end{aligned}$$

and

$$s_i^* = \begin{cases} 1 & \text{if } Y_i^* > Y_i^{**} \\ s & \text{if } Y_i^* = Y_i^{**} \end{cases}.$$

Similarly, we can summarize the set of parent first order conditions as

FOCs for Parents			
Cases		FOCs	
t_{jik}	H_i	H_i	t_{jik}
Int	Int	$\varepsilon_{X0} = T_0^H$	$\varepsilon_{tj0k} = T_{0jk}^{t3}(t_{j0k}, \varepsilon_{L0k})$
Int	Cor	$\varepsilon_{X0} \geq T_0^H$	$\varepsilon_{tj0k} = T_{0jk}^{t3}(t_{j0k}, \varepsilon_{L0k})$
Cor	Int	$\varepsilon_{X0} = T_0^H$	$\varepsilon_{tj0k} \leq T_{0jk}^{t3}(0, \varepsilon_{L0k})$
Cor	Cor	$\varepsilon_{X0} \geq T_0^H$	$\varepsilon_{tj0k} \leq T_{0jk}^{t3}(0, \varepsilon_{L0k})$

with

$$T_0^H = \frac{\beta_{10}\mu p_{X0} X_0 \bar{Q}}{\beta_{20}q}$$

$$T_{0jk}^{t3}(t_{j0k}, \varepsilon_{L0k}) = -\beta_{10}\tilde{\alpha}_{j0k} + \frac{\varepsilon_{L0k}\beta_{30k}}{L_{0k}} - \beta_{4j0k}.$$

Note that ε_{L0k} is an unnecessary error (in the sense that there is enough random variation to explain any observed event).

Define the set of first order conditions corresponding to solutions to FOCs as

$$\varepsilon = \varphi(\xi) \tag{9}$$

where ε is the vector of errors, ξ is the vector of endogenous variables, and $\varphi(\cdot)$ is the vector of functions implied by the first order conditions summarized above. We can use these first order conditions to construct a likelihood contribution for each family.¹⁶ For those elements of ξ corresponding to interior solutions, the relevant likelihood term is the density of the corresponding element of ε , and, for those elements of ξ corresponding to corner solutions, the relevant likelihood term is either the distribution function or one minus the distribution function of the corresponding element of ε , depending upon the nature of the corner solution. Equation (9) provides the set of first order conditions holding constant the behavior of all other family members. Thus, values of the error vector that satisfy equation (9) are consistent with the observed Nash equilibrium.

It is possible that the model has multiple equilibria for some families at some particular realizations of the ε vector. While we cannot rule out multiple equilibria analytically, we check for multiple equilibria at the estimates of the parameters separately for each family using a technique described in ES. Specifically, we consider multiple starting points for each family's optimal behavior in solving their equilibrium optimization problem and by checking for instability of any of the observed equilibria. We find no cases of multiple equilibria. Thus, while multiple equilibria are theoretically possible, they appear to be empirically unimportant.

¹⁶ Aguirregabiria and Mira (2007) use a similar approach in another context.

4.3 Nonlinear Budget Set Issues

The shape of the budget constraints pose several potential problems. First, equations (4) through (6) imply a kink in the children’s budget constraints where $Y^* = Y^{**}$ (because equation (4) depends on the $\max[Y^*, Y^{**}]$). The nonlinearity of the budget constraint could cause an endogeneity problem in the spirit of Hausman (1985) because the error vector ε that solves the first order conditions depends on observed endogenous choices. However, the likelihood function controls for this endogeneity.

A second problem may arise because equation (4) and the estimates of Y_i and s in equation (6) imply that the budget line is convex. In Figure 1, consider a child with an indifference curve like curve 1 who chooses the outcome at point A. Such a curve would correspond to a particular realization of the ε vector. However, if the child had a realization of the ε vector resulting in curve 2, any point between B and C would be preferable to point A. We need to rule out situations similar to curve 2 in Figure 1.

[INSERT FIGURE 1 HERE]

Third, we observe children at corner solutions. For these children, there must be no value of the errors satisfying the inequalities in the relevant first order conditions that cause the child to move to a different segment of the budget constraint. The leading case for such a problem is a child providing no financial help for formal care. This implies that ε_{Xi} must be greater than equation T_i^H . Theoretically, for large enough ε_{Xi} , the value of consumption would increase, possibly causing the child to move from a budget segment with low hours of work to one with high hours of work. However, as ε_{Xi} increases, ε_{Li} can increase to keep the child (and her spouse) on the observed budget segment.

We used the estimated parameter vector (displayed in Table 8) to measure the empirical importance of the second and third potential problem. For each child in each family at an interior solution, we computed the value of ε consistent with the observed choice. For each child in each family at a corner solution, we simulated 10 values of ε consistent with the observed choice. Conditional on ε , we allowed the child to optimize over all of her choice variables. We counted the number of times that the child chose something other than the observed choice. Over the 335,700 choices made, there were no deviations between observed choices and optimal choices conditional on ε . Thus, while there may be a theoretical problem caused by kinked budget sets, it is not an important problem empirically.

5 Data

We use the 1993 wave of the Assets and Health Dynamics Among the Oldest Old (AHEAD) data set to estimate our model. AHEAD is a nationally representative longitudinal data set designed to facilitate study of Americans aged

70 and older. Its emphasis on the joint dynamics of health, family characteristics, income, and wealth makes it a particularly rich source of information on families' decisions concerning care for elderly relatives, especially in light of its high response rate (over 80 percent). Although the 1993 wave contains only noninstitutionalized individuals, the exclusion of nursing home residents is not terribly problematic given our focus on informal and formal home health care. Moreover, although AHEAD oversamples blacks, Hispanics, and Florida residents, this oversampling causes no estimation bias because our analysis treats race/ethnicity and residential location as exogenous.

We use 3,583 of the 6,047 households in the first wave of the survey. As shown in Table 1, we excluded households for a variety of reasons. In most cases (1,116), records were missing data on the respondent, the respondent's spouse, or the respondent's children. Households with working respondents (270) or two respondents each of whom provided care for the other (25) were dropped to reduce the complexity of the model. Only the black and white non-Hispanic groups remained large enough for our analysis.

[INSERT TABLE 1 HERE]

Households included in AHEAD contain at least one respondent 70 years old or older. Many households also include spouses, some of whom are less than 70 years old. Spouses of respondents are also respondents. As a consequence of the exclusion of nursing home residents from the 1993 wave and the inclusion of spouses regardless of age, the characteristics of AHEAD respondents deviate from those of a representative individual who is 70 years old or older. The characteristics of respondents in our sample are shown in Table 2. On average, the male respondents (37 percent of the sample) are 76.7 years old with 11.7 years of education and 2.1 living children. Seventy-two percent are married, and 93 percent are white. On average, the female respondents are 76.3 years old with 11.8 years of education and 2.0 living children. Forty-two percent are married, and 90 percent are white.

[INSERT TABLE 2 HERE]

Twenty-three percent of men and 31 percent of women reported difficulty with an activity of daily living (ADL). The most common difficulty was walking across a room, reported by 17 percent of male respondents and 24 percent of female respondents. Thirteen percent of women and 8 percent of men reported difficulty bathing themselves, and prevalence rates for difficulty dressing were 12 percent among women and 10 percent among men. All other ADL problems had prevalence rates of less than 10 percent. Twenty-eight percent of women and 24 percent of men reported difficulty with an instrumental activity of daily living (IADL), most frequently difficulty with walking several blocks, pulling and lifting heavy objects, climbing stairs, or driving. The fraction of households reporting (paid or unpaid) help with an ADL or IADL in our sample is 22 percent. Of those households, 18 percent paid for care in the month prior to

the interview. The average amount paid per week among those paying for care is \$94.

Our measure of parental income includes income from major government transfer programs (e.g., Social Security, SSI, Food Stamps) and other nonwage income such as veteran’s benefits, retirement income, annuities, IRA distributions and income from stocks and bonds. A small number of respondents report positive wage earnings which we ignore so that we can avoid modeling the labor force behavior of the respondent. The average income of elderly households in our sample is \$417 per week. Most respondents were covered by Medicare and received assistance from the Supplemental Security Income program. Because the data do not include residents of nursing homes, few respondents reported eligibility for Medicaid.

[INSERT TABLE 3 HERE]

Table 3 contains information on the children of the respondents. Forty-nine percent of the children are male, and 70 percent are married. The average child is 47.0 years old with 14.0 years of education and two children. To model the decision-making process of the adult children of the elderly individuals, we need information on the market wages of the children. Since AHEAD does not provide these wages, we impute wages using the Current Population Survey by regressing log-wages on demographic characteristics available for the children of AHEAD respondents. Our estimates are reported in Table 4. The average imputed wage is \$452 per week. We also construct a measure of the leisure time consumed by the children and the respondents by treating time not spent working or caring for the parents as leisure.

As indicated in Table 3 and 4, respondents and their children experience a variety of living arrangements. Over half (55 percent) of respondents live with a spouse or an unmarried partner. Almost one fourth (23 percent) of respondent households include additional members; among these additional household members, 77 percent are children of the respondents. However, almost all children (94 percent) reside outside of the respondent’s household, and 62 percent of these children live more than 10 miles away.¹⁷

[INSERT TABLE 4 HERE]

Care arrangements also vary considerably across families. Table 5 displays patterns of caregiving in our sample. Overall 22 percent of elderly individuals receive formal or informal care in their homes. Among those receiving some type of care, 18 percent receive formal care, 90 percent receive informal care, and 8 percent receive both formal and informal care. Overall 6 percent of unmarried, childless respondents and 38 percent of married, childless respondents receive care in their homes. Regardless of the number of children, roughly one fourth of

¹⁷We observe whether the child lives with the parent, lives within 10 miles from the parent, or lives further than 10 miles of the parent. However, work such as Stern (1995) shows that marginal distance affects caregiving decisions only at greater distances. Thus, we do not use distance as a child characteristic.

elderly parents receive some type of care. Among families providing some type of care, the provision of informal care depends positively and the provision of formal care depends negatively on the number of adult children.

[INSERT TABLE 5 HERE]

Among elderly individuals receiving informal care, 63 percent receive care from their spouse, 42 percent receive care from their children, and 5 percent receive care from both their spouse and at least one of their children. Conditional on the receipt of informal care from at least one family member, the likelihood that the spouse and at least one adult child share informal caregiving responsibilities ranges from 3 percent of those with one child to 9 percent of those with five children. A more common type of shared caregiving involves two or more adult children. Among families with at least one informal care provider and at least two adult children, 14 percent include multiple caregivers among the younger generation. Not surprisingly, the likelihood that siblings share caregiving responsibilities depends positively on family size. Conditional on the receipt of informal care from at least one family member, 10 percent of elderly individuals with two children receive care from both children, while 17 percent, 19 percent, and 23 percent of elderly individuals with three, four, and five children, respectively, receive care from more than one child.¹⁸

Among families where elderly individuals receive formal home health care, 9 percent of elderly parents receive financial contributions for this care from their children. These results are consistent with other papers in the literature that show that financial contributions among family members are not that common.

These statistics understate the prevalence of informal and formal care, because only those AHEAD respondents reporting an ADL or IADL problem were asked about the provision of care. Furthermore, in the presence of an ADL or IADL problem, respondents were asked who provides care only if they reported receiving help with the problem “most of the time” and the amount of care is recorded only if the caregiver provided help at least once a week during the month prior to the survey. Thus, our measure does not capture sporadic care.

Moreover, these statistics understate the prevalence of multiple caregivers. In the case of ADLs, the respondent was asked only about the primary caregiver for each reported problem.¹⁹ Thus, respondents reporting a single ADL problem did not have the opportunity to report more than one caregiver. In the case of IADLs, the respondent was asked about the primary and secondary caregiver, if applicable, for a group of reported problems. Thus, while most families include a primary caregiver, multiple caregiving is an important phenomenon that deserves greater attention.

Table 6 displays the results of a probit model of informal care provision. The unit of observation is a potential care recipient – an elderly parent – combined with a potential caregiver – a child and/or child-in-law. The number of ADL

¹⁸The AHEAD survey does not provide detailed data for all children in families with more than five children.

¹⁹This bias is minor because care to assist with ADLs is the focus of the model.

problems is positively and significantly associated with the likelihood of informal care provision. In addition to problems with ADLs, several demographic characteristics are significantly associated with informal care provision in our probit model. The provision of care by a child or child-in-law depends positively on the parent’s age. Mothers are more likely than fathers and unmarried parents are more likely than married parents to receive informal care from a child or child-in-law. The parent’s education is negatively associated with the provision of informal care by adult children or children-in-law. Informal care provision is not significantly associated with child characteristics. Corresponding tobit models in Byrne, Goeree, Hiedemann, and Stern (2008) yield similar results concerning the quantity of care provided.

[INSERT TABLE 6 HERE]

The survey asks each parent whether or not she is “happy.”²⁰ Eighty-eight percent of parents reported being happy. We use the responses to this question to help identify some of the parameters in our structural model. Identification is discussed in more detail later. A probit model in Byrne, Goeree, Hiedemann, and Stern (2008) indicates that married individuals are more likely to respond affirmatively to this question than are unmarried individuals, men are more likely to respond affirmatively than are women, and whites are more likely to respond affirmatively than are blacks. Moreover, years of education are positively associated with happiness, while the number of ADL problems is negatively associated with happiness.

Finally, we construct a number of state-specific variables. These variables include a price level (Bureau of Economic Analysis 1999), the cost of home health care,²¹ and the average home health care state subsidy (US. Department of Health and Human Services 1992).

6 Estimation Strategy

6.1 Empirical Specification

To complete the specification of the model, we specify the variation of “parameters” across individuals within a family and the joint density of the errors. First, assume that α_{jik} in equation (1) is a function of parent and child characteristics,

$$\alpha_{jik} = \begin{cases} \exp \{ W_j^0 \delta_\alpha^* + W_k^0 \delta_\alpha^{**} \} & \text{if } i = 0 \\ \exp \{ W_j^0 \delta_\alpha^* + W_{ik} \delta_\alpha^{***} \} & \text{if } i > 0 \end{cases} \quad (10)$$

where W_j^0 is a vector of parent j ($j = m, p$) characteristics, W_k^0 is a vector of characteristics of the spouse (i.e., $k \neq j$), and W_{ik} is a vector of child characteristics for child i ($k = c$) and her spouse ($k = s$). Also, assume that

²⁰Specifically, the question reads, “Now think about the past week and the feelings you have experienced. Please answer yes or no to the following statements. ... (Much of the time during the past week) I was happy.”

²¹We used two sources of data, Census (1990) and Bureau of Labor Statistics (1998), to interpolate wages for home health aid workers in 1993.

$\log \mu$ is a constant, and the Z_j terms in equation (1) are functions of parent characteristics,

$$Z_j = \exp \{W_j^0 \delta_z\}. \quad (11)$$

Next, assume that, in equations (2) and (3), $\log \beta_{10}$, $\log \beta_{20}$, and β_{30k} are constant across families (with $\beta_{30k} = 1$), that $\log \beta_{1i}$ ($= \log \beta_{11}$), $\log \beta_{2i}$ ($= \log \beta_{21}$), and $\log \beta_{3ik}$ ($= \log \beta_{31}$) for $i > 0$ are constant across families and children within each family, and that

$$\beta_{4jik} = \begin{cases} W_{jk}^0 \delta_{\beta 4}^* + W_k^0 \delta_{\beta 4}^{**} & \text{if } i = 0 \\ W_{jik} \delta_{\beta 4}^{****} + W_{ik} \delta_{\beta 4}^{*****} & \text{if } i > 0 \end{cases}. \quad (12)$$

The terms β_{30k} and β_{4j0k} cannot be identified separately (except perhaps by functional form) because a parent's leisure time is determined jointly with her caregiving time. Thus, we set $\beta_{30k} = 1$ with no loss in generality. Also increasing the constant term in each β term simultaneously has no effect on the first order conditions. Thus, we set $\beta_{2i} = 1$. For the joint density of the errors, we assume

$$\begin{aligned} \varepsilon_{Xi} &= \exp \{ \eta_{Xi} \}, & (13) \\ \eta_{Xi} &\sim iidN(0, \sigma_{\eta X}^2), \\ \varepsilon_{Lik} &= \begin{cases} \exp \{ \eta_{Lik} \} & \text{for } i > 0 \\ 1 & \text{for } i = 0 \end{cases}, \\ \begin{pmatrix} \eta_{Lic} \\ \eta_{Lis} \end{pmatrix} &\sim iidN \left(0, \sigma_{\eta L}^2 \begin{pmatrix} 1 & \rho_L \\ \rho_L & 1 \end{pmatrix} \right), \\ \begin{pmatrix} \varepsilon_{tjic} \\ \varepsilon_{tjis} \end{pmatrix} &\sim iidN \left(0, \sigma_{\eta t}^2 \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \right), \\ \varepsilon_{tj0k} &\sim iidN(0, \sigma_{\eta t}^2) \text{ for } j \neq k = m, p, \\ \varepsilon_{ui} &\sim iidN(0, \sigma_u^2). \end{aligned}$$

Based on preliminary results and economic intuition, we restricted the effects of many parameters in order to estimate the effects of the explanatory variables. In general, we restricted a parameter using economic reasoning if, after controlling for the relevant actions, the characteristic would not be expected to influence the health production function or utility function in the manner indicated by the parameter. For example, we would not expect the education of the child to affect how much the child enjoys caring for her parent, after controlling for the amount of care provided; therefore we restrict the child education characteristic corresponding to the parameter $\delta_{\beta 4}^{****}$ (see equation 12). In contrast, the number of ADL problems experienced by the parent probably influences the parent's utility associated with caregiving; thus we do not restrict the number of ADLs characteristic corresponding to the parameter $\delta_{\beta 4}^*$. We cannot identify the constant terms in δ_{α}^* separately from δ_{α}^{**} or δ_{α}^{***} ; hence, we restrict the constant terms for δ_{α}^{**} and δ_{α}^{***} .

6.2 The Likelihood Function

The set of parameters to estimate is

$$\theta = (\delta_\alpha, \log \mu, \delta_z, \beta_0, \log \beta_{10}, \log \beta_{20}, \log \beta_{11}, \log \beta_{21}, \log \beta_{31}, \delta_{\beta 4}, \quad (14)$$

$$\gamma, \sigma_{\eta X}^2, \sigma_{\eta L}^2, \sigma_{\eta t}^2, \sigma_u^2, \rho_\eta, \rho_t),$$

and the set of data for observation $n = 1, 2, \dots, N$ is

$$\left\{ [t_{mik}, t_{pik}, L_{ik}, w_{ik}, W_i, a_{ik}]_{k \in c, s}, \widetilde{H}_i, Y_i, p_{Xi} \right\}_{i=1}^{I_n}$$

and

$$\left\{ t_{m0p}, t_{p0m}, \widetilde{H}_0, \overline{H}, u_0, Y_0, p_{X0}, q, W_m^0, W_p^0, a_{0p}, a_{0m} \right\}.$$

The variable t_{jik} is time spent caring for parent j by family member ik . As a result of data issues, we measure time in fractions of a week and we use a discrete measure of t_{jik} in computing the likelihood function. Its construction is discussed in the Appendix. The variable $\widetilde{H}_i = 1$ iff player i paid for care:²²

$$\widetilde{H}_i = 1 (H_i > 0).$$

The variable \overline{H} is the total amount of paid care:²³

$$\overline{H} = \sum_{i=0}^I H_i.$$

The variable

$$L_{ik} = 1 - \sum_{j \in m, p} t_{jik} - PT_{ik} \frac{20}{168} - FT_{ik} \frac{40}{168}$$

is leisure for family member ik where $PT_{ik} = 1$ iff child i (or child i 's spouse) works part-time and $FT_{ik} = 1$ iff child i (or child i 's spouse) works full-time. The variable w_{ik} is child i 's (or child i 's spouse) weekly wage. As discussed earlier, we estimate w_{ik} as a function of the observed characteristics of the child (or spouse) using a different data set. The variable Y_i is a measure of nonlabor income for player i . For the parent, Y_0 is observed. We assume that $Y_i = 0$ for $i > 0$. The variable p_{Xi} is the local price level for player i , and q is the price of care in the parent's state. The answer to the question about whether the parent considers herself happy, u_0 , is treated as a discrete measure of U_0 .²⁴ We do not observe a direct measure of health quality, Q . We observe the output of Q only indirectly through its effect on utility. W_{ik} are exogenous characteristics

²²The data do not provide enough information to actually determine if $\widetilde{H}_0 = 1$. We assume that, if paid care is provided, then some of it is paid for by the parents causing $\widetilde{H}_0 = 1$.

²³It is assumed that both parents, if alive, take advantage of paid care; i.e., that formal care is a public good for the parents' household.

²⁴The potential for observing two different "happy" answers for the husband and wife requires a small adjustment to the model, described in footnote 14.

for child i (or spouse), and W_m^0 and W_p^0 are exogenous parent characteristics. Define

$$\begin{aligned} t_{ji} &= \begin{cases} t_{jik} & \text{if } a_{ik} = 1, a_{il} = 0 \text{ for } l = c, s, l \neq k \\ (t_{jic}, t_{jis})' & \text{if } a_{ic} = a_{is} = 1 \end{cases}, \\ L_i &= \begin{cases} L_{ik} & \text{if } a_{ik} = 1, a_{il} = 0 \text{ for } l = c, s, l \neq k \\ (L_{ic}, L_{is})' & \text{if } a_{ic} = a_{is} = 1 \end{cases} \end{aligned}$$

for $i > 0$, and $t_i = (t_{mi}, t_{pi})$.

The likelihood contribution for family n , \mathcal{L}_n , is a product of conditional probabilities over different events (such as whether or not the child contributes time or financial resources to care for the parent). Its structure varies with characteristics of the family's choices and can be written as

$$\begin{aligned} \mathcal{L}_n &= \left\{ \Pr[u_0 \mid \widetilde{H}_0, t_0] \prod_{\substack{j \in m, p \\ k \neq j}} \Pr[t_{j0k}]^{a_{0k} a_{0j}} \right\} \bullet \\ &\quad \prod_{i: \widetilde{H}_i = 0} \left\{ \int_{\eta_{Xi} \geq \ln T_i^H} \Pr[t_i, L_i \mid \widetilde{H}_i = 0, \varepsilon_{Xi}]^{1(i>0)} \frac{1}{\sigma_{\eta X}} \phi \left[\frac{\eta_{Xi}}{\sigma_{\eta X}} \right] d\eta_{Xi} \right\} \bullet \\ &\quad \iiint_{\substack{\eta_{Xi} \leq \ln T_i^H \\ i: \widetilde{H}_i = 1}} 1 \left(\sum_{i: \widetilde{H}_i = 1} H_i(\eta_{Xi}) = \overline{H} \right) \prod_{i: \widetilde{H}_i = 1} \Pr[t_i, L_i \mid \widetilde{H}_i = 1]^{1(i>0)} \bullet \\ &\quad \frac{1}{\sigma_{\eta X}} \phi \left[\frac{\eta_{Xi}}{\sigma_{\eta X}} \right] d\eta_{Xi} \end{aligned} \quad (15)$$

where

$$H_i(\eta_{Xi}) = \frac{1}{q} \left(Y_i + \sum_k w_{ik} \left(1 - L_{ik} - \sum_{j \in m, p} t_{jik} \right) - \frac{\beta_{2i} q}{\beta_{1i} \mu \overline{Q}} \exp\{\eta_{Xi}\} \right) \quad (16)$$

is derived from equations (8) and the set of first order conditions for the children. The $\Pr[u_0 \mid \widetilde{H}_0, t_0]$, $\Pr[t_{j0k}]$, $\Pr[t_i, L_i \mid \widetilde{H}_i = 0, \varepsilon_{Xi}]$, and $\Pr[t_i, L_i \mid \widetilde{H}_i = 1]$ terms are defined in Appendix S1 in Byrne, Goeree, Hiedemann and Stern (2008).

Some of the terms in the likelihood function need to be simulated. With the exception of the last term simulation, is straightforward. We discuss how to simulate the last term using a GHK algorithm (Hajivassiliou, McFadden, and Ruud 1996) in Appendix S2 in Byrne, Goeree, Hiedemann and Stern (2008).

6.3 Identification

The set of parameters to estimate is listed in equation (14). Asymptotically, we can observe consistently the covariation of each dependent variable with the set of exogenous variables. It is this covariation that allows us to identify all of the structural parameters.²⁵ The effect of parent exogenous variables on baseline health Z , measured by δ_z in equation (11), is identified by covariation between parent characteristics and the “happy variable,” u_0 . The effects of exogenous variables on parent and child utility, measured by δ_{β_4} in equation (12) and $\log \beta_{10}$, $\log \beta_{20}$, $\log \beta_{11}$, $\log \beta_{21}$, and $\log \beta_{31}$, are identified by covariation between parent and child characteristics and parent and child choices. For example, the degree that the parent’s problems with ADLs move with child informal care t identifies the effect of parent ADLs on β_4 . Covariation between parent characteristics and children’s care decisions does not identify δ_z because parent characteristics can directly affect care decisions through δ_{β_4} . The effect of parent and child characteristics on the quality of care, measured by δ_α in equation (10), is identified by the covariation between $\partial \Pr[u_0 = 1 | t] / \partial t$ and parent and child characteristics. For example, because the partial correlation between parent happiness and informal care provision increases with the age of the child caregiver, the δ_α coefficient on child age is positive. Thus, we should observe that the sample slope of $\Pr[u_0 = 1 | t]$ with respect to t varies with age appropriately. Even when there are no covariates affecting α and μ , we can identify α and μ by the covariation of the “happy” variable with the receipt of informal and formal care, respectively. The term γ in equation (1) is identified by $\partial^2 \Pr[u_0 = 1 | t] / \partial t^2$. The effectiveness of formal care, measured by $\log \mu$, is identified by the covariation between u_0 and the provision of formal care, \bar{H} . The term β_0 in equations (2) and (3) is not of interest by itself. But it is needed to match the mean of the “happy variable” data and is identified by the mean. Second moment terms, $\sigma_{\eta X}^2$, $\sigma_{\eta L}^2$, $\sigma_{\eta t}^2$, σ_u^2 , ρ_η , and ρ_t , are identified by variances and correlations of generalized residuals (Gourieroux, et al. 1987) associated with the likelihood function.

The provision of informal care t affects a family member’s utility in two ways: it directly affects utility through the satisfaction (or sense of burden) one receives (the β_4 effect), and it improves the parent’s health quality, thus affecting the child’s utility (the β_1 effect). Most of the literature on informal care does not specify which mechanism is relevant. In almost all of the literature, there is no attempt to identify the two effects separately. HS argue that all children derive utility from the health benefits of informal care but only the caregiver derives satisfaction or burden from it. Thus, HS identify the separate effects by variation in care provision across families of different sizes. We are making the same assumption, but the effect of informal care on identification is completely different because the games being played in the two models are very different. As mentioned previously, we do not observe direct measures of health quality

²⁵Of course, there is the possibility of singularity not considered in this discussion. The final argument for identification is empirical: the Hessian of the log likelihood function is non-singular.

(Q), but this is not necessary for identification. In this work, the inclusion of the “happy variable” allows us to directly measure the effect of formal and informal care on the parent’s well-being, and that allows us to disentangle the two effects. For example, in the presence informal care provision by some children, if we were to observe that the provision of informal care by children has a very small empirical effect on the parent’s happiness relative to the effect of variables affecting Z , we would conclude that α is very small and $\beta_4 > 0$. Alternatively, if we were to observe that very little informal care is provided but those parents who receive it are usually happier, we would conclude that α is large and $\beta_4 < 0$. The inclusion of the “happy data” allows us to nonparametrically identify all of the parameters because terms like $\partial \Pr[u_0 = 1 | t] / \partial t$ and its covariation with observed variables are nonparametrically identified. The model structure tells us how to decompose $\partial \Pr[u_0 = 1 | t] / \partial t$ into $\partial \Pr[u_0 = 1 | t] / \partial Q$ and $\partial Q / \partial t$, but the model works regardless of the decomposition.

7 Results

7.1 Parameter Estimates

Our structural model captures several dimensions of families’ caregiving decisions including the effectiveness of formal and informal care, the burden associated with informal care provision, and altruism toward elderly spouses and parents. Our model allows us to identify parameters related to these dimensions of the decision-making process even in the absence of independent variables. Table 7 displays the results of a specification that does not depend on family members’ observed characteristics. Despite its relative simplicity, the estimates of this specification shed light on the motivations for the provision of formal and informal care. Our health quality production function in equation (1) implies constant returns to formal care but allows for increasing, constant, or decreasing returns to informal care. In the absence of covariates, the estimates suggest that both formal and informal care have modest effects on the parent’s health quality and that there are diseconomies of scale associated with informal care. Moreover, the estimates suggests that not only is informal care relatively ineffective but its provision tends to be burdensome. These results may explain why few family members provide care for elderly individuals. However, the results of this simple version of the model imply that adult children and children-in-law care about their parents’ health quality, suggesting that altruism may play an important role in the provision of informal and formal care.

[INSERT TABLE 7 HERE]

Another version of our model allows the α and Z terms in equation (1) and the β_4 term in equations (2) and (3) to depend on covariates. This specification allows family members’ characteristics to affect both the quality of care provided and the burden associated with caregiving. In addition, this specification allows elderly individuals’ characteristics to affect their health quality

and, in turn, all family members' utility. A child or child-in-law's provision of care depends on the parent's health quality, the effectiveness of informal care, and the burden associated with caregiving. An elderly individual's utility depends on the effectiveness of care but not on her children's caregiving burden. Thus, we can identify the effect of characteristics such as the parent's age on the burden associated with informal care provision from the effect of the same characteristic on the quality of informal care. Table 8 presents the results of our model with covariates, and Table 9 displays the first two moments of relevant model characteristics. A likelihood ratio test rejects the model without covariates in favor of the model with covariates.

[INSERT TABLE 8 HERE]

Gender Our structural model provides three distinct mechanisms for an elderly individual's gender to influence the provision of informal care. Specifically, our model allows for the possibility that health quality, the effectiveness of informal care, and the burden associated with its provision differ for elderly men and women. Controlling for age, race, marital status, and the number of ADL problems, our results suggest that elderly men experience significantly greater health quality than do elderly women prior to any formal or informal care decisions ($Z_{\text{mother}} = -1.630$). Thus, the marginal utility associated with the mother's health quality exceeds the marginal utility associated with the father's health quality. In turn, children face greater incentives to provide care for mothers than for fathers, abstracting from the effects of gender on the quality of care and the burden associated with its provision. Moreover, our results suggest that informal care provided to mothers (wives) is significantly less burdensome than care provided to fathers (husbands) ($\beta_{4\text{mother}} = 0.584$), again providing children with greater incentive to spend time caring for elderly mothers than fathers. Similarly, our probit results of informal care provision indicate that mothers are significantly more likely than fathers to receive informal care from children or children-in-law, and HS report that families value care provided for mothers more than care provided for fathers. However, the results of our structural model suggest that informal care provided to mothers (wives) is significantly less effective ($\alpha_{\text{mother}} = -1.438$) than informal care provided to fathers (husbands). This gender difference may shed light on PS's finding that daughters are more likely to provide care for fathers than mothers. Overall the results of our structural model suggest that elderly women may have greater caregiving needs than do elderly men; although care provided to mothers is significantly less burdensome than care provided to fathers, it is significantly less effective. The complex relationship between gender and motives for informal care provision may contribute to the conflicting evidence presented in the literature.

Child gender also plays a role in family caregiving. ES, CS, and SPH find that, all else equal, daughters are significantly more likely than sons to provide care, whereas SPH's findings indicate that sons provide significantly more care than do daughters. Our structural model allows both the effectiveness

of informal care and the burden associated with its provision to differ by child gender. The model also distinguishes between children and children-in-law. In our raw data, 7.0 percent of daughters provide informal care, compared to 4.0 percent of sons, 1.6 percent of daughters-in-law, and 0.8 percent of sons-in-law. These differences suggest that the quality of care, the burden associated with its provision, and/or opportunity costs may differ by gender; similarly, children may provide higher quality care and experience less burden than their spouses. In fact, the results of our structural model indicate that children provide higher quality care and experience less burden than do children-in-law. In particular, daughters provide the highest quality care ($\alpha_{\text{biological}} = 0.331$) and experience the least burden ($\beta_{4\text{biological}} = 0.431$) followed by sons ($\alpha_{\text{male}} + \alpha_{\text{biological}} + \alpha_{\text{male*biological}} = 0.319$; $\beta_{4\text{male}} + \beta_{4\text{biological}} + \beta_{4\text{male*biological}} = 0.120$), then daughters-in-law (reference category), and finally sons-in-law ($\alpha_{\text{male}} = -0.323$; $\beta_{4\text{male}} = -0.269$).²⁶

Aging and the Development of ADL Problems The existing literature presents evidence that children provide more informal care as parents age (e.g., CH and PS) or develop more ADL problems (e.g., SPH and CS). Similarly, our probit model indicates that the provision of care by a child or child-in-law depends positively on the parent’s age and on the number of ADL problems experienced by the parent. As in the case of the elderly individual’s gender, our structural model provides three distinct mechanisms for an elderly individual’s age and ADL problems to influence the provision of informal care. Age and ADL problems may influence health quality, the effectiveness of informal care, and the burden experienced by family members in the provision of care. Controlling for ADL problems, informal care becomes significantly more effective ($\alpha_{\text{age}} = 0.12$) and significantly less burdensome as parents grow older ($\beta_{4\text{age}} = 0.04$). Thus, abstracting from the effects of aging on health quality, family members face greater incentives to provide informal care as elderly individuals age. As an elderly individual accumulates ADL problems, her health quality declines insignificantly ($Z_{\text{ADL}} = -0.184$) while informal care becomes insignificantly less effective ($\alpha_{\text{ADL}} = -0.023$) and significantly more burdensome ($\beta_{4\text{ADL}} = -0.172$). Thus, problems with ADLs have conflicting effects on family members’ incentives to provide informal care.

Our results reveal surprising patterns concerning aging and health quality. Controlling for ADL problems, an elderly individual’s health quality depends positively and significantly on her age ($Z_{\text{age}} = 0.07$). This counterintuitive result may be an artifact of the sample selection procedure. As mentioned earlier, the 1993 wave of the AHEAD data contains only noninstitutionalized elderly individuals. Thus, the oldest members of the sample may be particularly healthy for their age, controlling for ADL problems. In fact, a few of the older respondents in our sample have large consumption expenditures but receive no informal or

²⁶Even though many of the relevant estimates are not statistically significantly different from zero, $H_0 : \alpha_{\text{biological}} = \alpha_{\text{male}} + \alpha_{\text{biological}} + \alpha_{\text{biological*male}} = \alpha_{\text{male}} = 0$ is rejected at the 10 percent significance level, and $H_0 : \beta_{4\text{biological}} = \beta_{4\text{male}} + \beta_{4\text{biological}} + \beta_{4\text{biological*male}} = \beta_{4\text{male}} = 0$ is rejected at the 5 percent significance level.

formal care. If aging reduced their health quality, these respondents probably would have reallocated some of their consumption expenditures to formal care or their family would have reallocated some of its leisure time to informal care provision. In the absence of these outliers, the effect of age on health quality would be negative. As illustrated by these outliers, choices concerning consumption and leisure conditional on income provide information about the need for formal and informal care. Overall, the results imply that the marginal utility associated with the individual’s health depends negatively on her age. Thus, our results, albeit influenced by a few outliers, suggest that family members’ incentives to provide care diminish as parents age. Recall, however, that these implications abstract from the effects of aging on the effectiveness of informal care and the burden associated with its provision.

Our model also allows for the spouse’s age and ADL problems to influence the effectiveness of informal care. Neither of these relationships approaches statistical significance.

Children’s Ages and Parity In addition, our model allows for the age of a child and her parity (whether she is the oldest child) to influence the effectiveness of informal care and the burden associated with its provision. Consistent with the results presented in HS, neither the age of a child nor her parity is significantly associated with the provision of informal care in our probit model. Our structural model reveals a more complex relationship between a child’s age and the provision of informal care, namely that children provide significantly less effective care ($\alpha_{\text{childage}} = -.04$) but experience significantly less burden ($\beta_{4\text{childage}} = 0.06$) as they age. The reduction in quality may be attributable to diminished health and energy of children as they age, while the reduction in burden may be attributable to the reduced demands on adult children’s time as their own children reach adulthood and leave home. (Our model controls for an adult child’s family size but not the ages of her children.) Controlling for age, our structural model indicates that oldest children provide significantly more effective care ($\alpha_{\text{oldest}} = 0.218$) but experience significantly greater burden ($\beta_{4\text{oldest}} = -0.245$) than their siblings. Thus, an adult child’s age and parity both have ambiguous effects on her incentives to provide informal care.

Marriage and Family Size An elderly individual’s marital status influences the family’s care decisions. Consistent with other studies (e.g., HS, ES, CS, and PS), our probit model indicates that married individuals are less likely to receive informal care from their children or children-in-law than are unmarried individuals. This result suggests that marriage enhances health and/or that married individuals are more likely to rely on their spouses than on their children for the provision of care. Our structural model provides support for both of these explanations. Married individuals enjoy significantly greater health than do their unmarried counterparts prior to any formal or informal care decisions ($Z_{\text{married}} = 0.482$). The model does not directly allow for the possibility that an elderly individual’s marital status influences the effectiveness of informal

care or the burden associated with its provision. However, the model allows for the quality and burden associated with informal care to differ for spouses and children. While, on average, children are more effective caregivers than are spouses (the mean $\log \alpha$ is greater for children than for spouses), they tend to experience greater burden (the mean β_4 is almost 36 times larger for children than for spouses). This discrepancy in caregiving burden contributes to spouses' greater propensity to provide care. For example, our parameter estimates indicate that, in about 80 percent of families with a married elderly individual and one adult child, the elderly individual's spouse is more likely than her child and/or child-in-law to provide care.

Adult children's family structure may also influence care decisions. Our structural model allows for an adult child's marital status and family size to influence both the effectiveness of care and the burden associated with its provision. Consistent with SPH's finding that married children provide less care for their elderly parents than do unmarried children, our results indicate that married children provide significantly less effective care ($\alpha_{\text{married}} = 0.154$) than their unmarried counterparts. However, marital status is not significantly related to caregiving burden. Surprisingly, an adult child's family size is not significantly related to her caregiving effectiveness or burden.

Education Previous studies (e.g., HS and ES) use education as a proxy for income and attribute the greater reliance on formal care rather than informal care among more highly educated individuals to an income effect. Similarly, our probit model indicates that parental education is negatively associated with informal care provision by children or children-in-law. In our structural model, family members make caregiving decisions as part of a broader constrained maximization problem. Thus, our model and data disentangle the effects of income and education. Specifically, our model allows for the possibility that the effectiveness of informal care depends on the education of both the elderly parent and the child, while the burden associated with care provision depends on the education of the child. The results indicate that the effectiveness of care depends negatively and significantly on the parent's education ($\alpha_{\text{education}} = -0.012$). Thus, all else equal, family members have less incentive to spend time caring for more highly educated parents. This result suggests that the greater reliance on formal care among more highly educated elderly individuals may be attributed to differences in informal care effectiveness rather than income, thus highlighting the importance of isolating the effects of income and education.

Controlling for opportunity costs, adult children's education is not significantly related to their caregiving effectiveness or burden. However, as demonstrated in Table 4, adult children's opportunity costs depend positively and significantly on their educational attainment. Thus, education influences informal care provision indirectly through its impact on opportunity costs. In the next section, we examine the role of opportunity costs in children's informal care decisions.

Race Other studies provide mixed evidence concerning the role of race in family caregiving (Wolf 1984, Spear and Avery 1993, Stern 1995). In our sample, black parents are significantly more likely than white parents to receive informal care from their children or children-in-law. Specifically, 15.6 percent of black parents receive informal care from children and/or children-in-law compared to 2.7 percent of white parents, a difference that is statistically significant at the 0.1 percent level. Also, among caregivers, black children and their spouses provide significantly more care than do white children and their spouses: 25.0 versus 15.6 hours per week, a difference that is significant at the 5 percent level. However, patterns differ for mothers and fathers: black mothers are significantly more likely than white mothers to receive informal care from children and/or children-in-law, while black fathers are insignificantly less likely than white fathers to receive informal care from children or children-in-law.

Descriptive statistics by race reveal several significant differences. For example, white respondents are more highly educated on average than are black respondents. Also, white respondents are significantly more likely to be married than are black respondents. As discussed above, parental education and marriage are negatively associated with informal care provision by children. Thus, the question arises as to whether informal care provision varies by race after controlling for relevant characteristics that are correlated with race.

Race is not significantly related to the provision of informal care in our probit model. As in the case of the elderly individual's age, ADL problems, and gender, our structural model allows for the possibility that health quality, the effectiveness of informal care, and the burden associated with its provision depend on the race of the elderly individual. Race does not significantly influence the quality of care, the burden associated with caregiving, or the parent's health. Significant differences in care propensities between white and black families observed in the raw data thus reflect racial differences in other characteristics such as marital status and education.

Motivations for the Provision of Formal and Informal Care As discussed above, the second specification allows for observed and unobserved variation within and across families regarding the effectiveness of informal care and the burden associated with its provision (as well as opportunity costs and health quality). These sources of variation shed light on the motivations for the provision of informal care and the varied patterns observed in the data. The model also enables us to estimate the effect of formal care on elderly individuals' health quality and the extent of altruism toward elderly parents and spouses. These parameter estimates further illuminate motives for informal and formal care provision.

[INSERT TABLE 9 HERE]

Again the results reveal modest effects of formal care on elderly individuals' health quality. As indicated in Table 9, results of our specification with covariates imply that formal care provided around the clock (24 hours a day,

seven days a week) would have virtually no effect on the average elderly individual’s health quality $[(\partial(\log\text{Health Quality})/\partial(\text{Formal Care}))= 0.000]$ relative to no formal care. Thus, as indicated in Table 10, our estimates generate low predicted probabilities of the use of formal care. The discrepancy between the actual rates of formal care use reported in Table 5 and the predicted probabilities reported in Table 10 suggests that our model may not be fully capturing the benefits of formal care.

[INSERT TABLE 10 HERE]

The results also reveal modest effects of informal care. Informal care provided by an adult child tends to be more effective than formal care. On average, an additional hour of informal care provided by an adult child is associated with a 0.12 percent increase in health quality $[(\partial(\log\text{Health Quality})/\partial(\text{Informal Care}))/168 = 0.0012]$, while an additional hour of informal care provided by a spouse has virtually no effect on the elderly individual’s health quality. Here the estimated γ indicates that the first 98.5 hours of informal care provided by a particular family member in a week enhance the elderly individual’s health quality.

Although children provide higher quality care, on average, than do spouses, our results suggest that children generally experience greater burden in the provision of care. The component of the caregiving burden that depends on observed characteristics is almost 36 times greater, on average, for children than for spouses. Our model does a reasonable job of predicting hours of care for children. But it significantly overpredicts informal care provided by spouses. This discrepancy could be avoided by the addition of either a fixed burden cost in providing informal care or a term like γ in the burden specification. Namely, the last term in the utility function in equation (2) would change to either

$$(\beta_4 + \varepsilon_t)t + \xi 1(t > 0) \quad \text{or} \quad (\beta_4 + \varepsilon_t)t + \xi^2. \quad (17)$$

Given the present specification of the model, α and γ allow us to fit both the informal care participation decision (i.e., whether $t > 0$) and the hours decision for children; essentially, α determines the participation decision and γ adjusts to match the hours choice. This same mechanism, however, does not work for spouses of parents. For them, since the value of α is so small, all of the variation in parental informal care participation comes from variation in β_4 . Since there is no nonlinear term like γ associated with burden such as ξ in equations (17), the model cannot match both features of the data for informal care provided by parents. One might ask why α for spouses does not adjust to allow γ to affect their decisions. This is prevented by the small covariation between parent happiness and informal care provided by a spouse.

As in the model without covariates, the results of the model with covariates imply that adult children and children-in-law care about their parents’ health quality, suggesting that altruism may play an important role in the provision of informal and formal care. For an adult child with average characteristics, a one percent increase in the child’s consumption increases the child’s utility by

less than half as much as a one percent increase in an elderly parent’s health quality: the mean $(\partial U/\partial \log X)/(\partial U/\partial \log Q) = 0.49$. Similarly, a one percent increase in the average child’s leisure increases the child’s utility by about 61 percent as much as a one percent increase in the elderly parent’s health quality: the mean $(\partial U/\partial \log L)/(\partial U/\partial \log Q) = 0.61$. Moreover, adult children derive more utility than do elderly individuals and their spouses from a one percent increase in the elderly individual’s health quality: the mean $\partial U/\partial \log Q$ is 3.91 for adult children and 1.85 for elderly individuals.

Collectively these patterns suggest that the greater propensity of spouses than children to provide informal care is attributable to their lower burden – perhaps partially in the form of fixed costs – in the provision of care rather than differences in care effectiveness between spouses and children or selfishness on the part of adult children.

Motivations for Shared Caregiving In our sample, shared caregiving is relatively uncommon. For example, as indicated in Table 5, only 14.4 percent of families with at least one adult child providing care exhibit shared caregiving among adult siblings. Similarly, as shown in Table 10, estimates of our structural model imply that the predicted probability of shared caregiving among such families is 12.6 percent.

Although most elderly individuals rely on a single caregiver, shared caregiving has the potential to alleviate caregiver burden and reduce the likelihood of caregiver burnout. Also, as discussed earlier, a bequest motive may motivate children to provide care, especially in the presence of care provided by a sibling, as a way to compete for a greater share of the inheritance. Thus, it is worth exploring factors that influence whether adult children share caregiving responsibilities. Our structural model enables us to examine several possible reasons for the infrequency of shared caregiving among adult children: increasing returns to informal care, free-riding, ineffective care, and caregiving burden. The first explanation concerns the health quality production function. This production function allows for increasing, constant, or decreasing returns to informal care provided by a particular family member. While increasing returns would encourage the reliance on a single caregiver, decreasing returns would encourage shared caregiving. In light of strong evidence of decreasing returns to informal care provided by a particular family member (γ is significantly less than 0), concerns about the quality of care do not explain the infrequency of shared caregiving.

The second explanation for the lack of shared caregiving concerns the possibility of free-riding. The infrequency of shared caregiving could be attributable to free riding if adult siblings’ informal care decisions were negatively correlated (see CS for a discussion of the issue). However, the simulated informal care decisions of children are nearly independent of one another, suggesting that free-riding is not prevalent.²⁷ The low correlations between siblings’ care prob-

²⁷Using a hypothesis test described in Checkovich and Stern (2002), we simulated each family 10 times with antithetic acceleration (for a total of 20 times per family) and then

abilities occur because other phenomena such as elderly parents’ preferences for shared caregiving counteract adult children’s incentives to free-ride on one another. More technically, the latent utility of an elderly parent is concave with respect to the informal care provided by individual children (as measured by the correlation of the “happy” variable with informal care provided by a particular child).²⁸

The third and fourth explanations for the lack of shared caregiving concern children’s motivations to provide care in terms of their effectiveness in the caregiving role, as measured by α and γ , and the burden experienced in the provision of care, as measured by $\beta_4 + \varepsilon_t$. The estimate of γ and the distributions of α and $\beta_4 + \varepsilon_t$ imply low probabilities that a particular child provides informal care. As mentioned above, on average, an additional hour of informal care provided by a particular child is associated with a 0.12 percent increase in the elderly parent’s health quality. The modest returns to informal care afford children with little motivation to provide care, especially in light of the burden associated with caregiving. Our results suggest that the majority – 97.3 percent – of adult children consider caregiving burdensome. Collectively, these results suggest that individual children have little incentive to provide informal care for elderly parents. Moreover, as discussed above, the simulated informal care decisions of children approach independence. Thus, the probability that two (or more) adult siblings share caregiving responsibilities may be approximated by the product of two (or more) small probabilities.

A final explanation for the lack of shared caregiving, not captured by our structural model, concerns the fixed costs of informal care provision. The presence of fixed costs including travel time to an elderly parent’s home may contribute to the infrequency of shared caregiving observed in the raw data. However, the similarity of the actual and predicted probabilities of shared caregiving reported in Tables 5 and 10, respectively, indicate that the omission of fixed costs from the model does not severely limit the model’s ability to predict shared caregiving. As discussed above, the omission of fixed costs may have greater implications for our estimated intergenerational differences in caregiving burden.

7.2 Decompositions

Differences in opportunity costs, caregiving effectiveness, and caregiving burden between sons and daughters contribute to daughters’ greater likelihood of providing informal care for elderly parents. In order to better understand the relative importance of these three factors, we examine their separate and joint effects on adult children’s likelihood of providing informal care. In addition to

computed the Nash equilibrium. Although the resulting $\hat{\rho}_j$ statistics are significant for families with two, four, or five children, none would be significant if the number of observations were the same as in the relevant subsample. Thus, based on the actual sample sizes, we cannot reject the null hypothesis that adult children’s behavior is independent.

²⁸In effect, if we were to estimate a probit model of the “happy” variable that included an individual child’s informal care in quadratic form, the coefficient on the linear term would be positive and that on the quadratic term would be negative.

the overall probability that an adult child provides informal care conditional on the child’s gender, number of siblings, and marital status, Table 11 displays corresponding conditional probabilities that isolate the effects of opportunity costs, quality of care, and caregiving burden. For all family sizes, the benchmark is an unmarried daughter. For example, consider families that consist of only one unmarried adult child. As indicated in the first column, the log probability of informal care provision is -3.864 for daughters and -5.735 for sons. Thus, the estimated probability of informal care provision is 2.1 percent for daughters and 0.3 percent for sons. Each of the next three columns removes gender differences in two of the three following dimensions: 1) opportunity costs as measured by wages, 2) caregiving effectiveness or quality, and/or 3) caregiving burden. Allowing for gender differences in wages but not in quality or burden, the log probability that an unmarried son with no siblings provides care is -4.079 , compared to -3.864 for an unmarried daughter with no siblings. Removing the gender differences in caregiving effectiveness and burden thus closes much of the gap: the estimated probability of care provision would be 1.7 percent for sons compared to 2.1 percent for daughters if sons provided as effective care and experienced as little burden as daughters. Allowing for gender differences in only quality or only burden, the log probability that an unmarried son with no siblings provides care is -4.960 or -4.454 , respectively. The decompositions suggest that differences in opportunity costs, quality, and burden account for much of the gender gap in informal care provision. For unmarried sons without siblings, the estimated probability of care provision – 1.9 percent – would approach the corresponding figure for daughters – 2.1 percent – if sons had the same opportunity costs, caregiving effectiveness, and caregiving burden as daughters. Interestingly, in families with three or more children, unmarried sons would display higher probabilities of informal care provision than daughters if their care were as effective and if they experienced as little burden as daughters.

[INSERT TABLE 11 HERE]

Differences in opportunity costs, caregiving effectiveness, and caregiving burden between married and unmarried children may also contribute to different caregiving propensities by marital status. Our estimated wage equation indicates that married men earn more than unmarried men, while our structural model suggests that marriage is negatively associated with the quality of care and insignificantly associated with caregiver burden. However, the presence of a second potential caregiver may counteract differences in quality and opportunity costs. Further complicating the situation, daughters provide more effective care and experience less burden than do sons but their husbands provide less effective care and experience more burden than do sons’ wives. The decompositions shown in Table 11 shed light on differences by marital status and gender. In the case of married children, Table 11 provides the log probability that married daughters and/or their husbands provide informal care and the log probability that married sons and/or their wives provide care. For all family sizes, married daughters display lower overall care probabilities than their unmarried counterparts. For example, in the absence of siblings, married daughters and/or

their husbands display care probabilities of 0.7 percent compared to 2.1 percent for unmarried daughters. In families with three or fewer children, this pattern persists even in the absence of differences in opportunity costs, quality, and/or burden. As family size increases, this pattern generally reverses when one effect is considered at a time. For example, consider the effects of isolating quality differences by marital status in families with four children. If the care provided by married daughters and their husbands were as burdensome as the care provided by unmarried daughters and if married daughters and their husbands faced the same opportunity costs of time as unmarried daughters, then married daughters and/or their husbands would display slightly higher care probabilities than unmarried daughters. With two exceptions – families with three or five children – married sons and/or their wives tend to display higher care probabilities than unmarried men. Overall daughters and their husbands are more likely to provide care than are sons and their wives. However, in the absence of quality differences, the opposite pattern would prevail, namely that sons and their wives would be more likely than daughters and their husbands to provide informal care.

While the presence of siblings may motivate adult children to free-ride on one another and thus reduce caregiving incentives, the possibility of a bequest may motivate children to compete with one another for an inheritance. Holding child gender and marital status constant, the probability that a child provides informal care generally decreases as the number of siblings increases. Thus, the results in Table 11 provide evidence against the bequest motive. Although this pattern is consistent with free-riding behavior, the near independence of siblings' informal care probabilities (see the discussion above) casts doubt on the importance of this phenomenon.

Taking the decompositions a step further illustrates the extent to which opportunity costs, caregiving effectiveness, and caregiving burden contribute to gender differences in the propensity to provide informal care. Table 12 reports cross partial differences of log probabilities with respect to the effect in question and gender. Consider, for example, families with one unmarried adult child. As indicated in Table 12, opportunity costs reduce the probability that a son provides informal care by 12 percent ($\exp(-0.128) - 1 = -0.120$) relative to the effect of opportunity costs for a daughter. Quality and burden effects reduce the probability that a son provides informal care by 64 and 40 percent, respectively, relative to the same effects for a daughter. The results are reasonably robust to changes in family size.

[INSERT TABLE 12 HERE]

Now consider married children. Again start by considering families with one child. Overall sons and/or their wives are 60 percent less likely to provide care than are daughters and/or their husbands. Wage, quality, and burden effects reduce the probability that a son and/or daughter-in-law provide informal care by 13, 40, and 16 percent, respectively, relative to the same effects for a daughter and/or son-in-law. These effects tend to increase as family size increases.

Racial differences in adult children’s opportunity costs, caregiving effectiveness, and caregiving burden as well as elderly parents’ health quality contribute to or offset the greater prevalence of informal care provision in black relative to white families. Our wage equation indicates that whites, especially married white men, earn more than blacks. Our structural model suggests that white parents may experience greater health quality than do black parents. Although this health difference is not statistically significant, it may indicate greater caregiving needs on the part of black parents. However, the results of our structural model also indicate that white children provide insignificantly higher quality care and experience insignificantly less burden than do black children.

The decompositions reported in Table 13 illustrate the extent to which opportunity costs, caregiving effectiveness, and caregiving burden influence racial differences in the propensity to provide informal care. In the raw data, black parents display significantly higher probabilities of receiving informal care, but the decompositions suggests that white children are more likely to provide care after controlling for relevant demographic characteristics. For example, the results in Table 13 indicate that among unmarried only children, whites are 34 percent more likely to provide care than are blacks. Quality and burden effects increase the probability that a white unmarried only child provides care by 37 and 4 percent, respectively, relative to the same effect for a black unmarried only child. Operating in the opposite direction, wage effects reduce the probability that a white unmarried only child provides care by 6 percent relative to the same effect for a black unmarried only child. Among married children without siblings, white children and their spouses are 20 percent more likely to provide care than are black children and their spouses. The burden effects are similar for married children, but the wage effects are larger and the quality effects are smaller. For both unmarried and married children, these results are relatively robust to changes in family size. Finally, racial differences in health quality, though not statistically significant, may contribute to the patterns observed in the raw data.

[INSERT TABLE 13 HERE]

7.3 Specification Tests

We performed two types of specification tests. First, we tested for the existence of state fixed effects. We aggregated residuals for 34 states with at least four observations. We could not reject the null hypothesis of no state fixed effects for time spent caring for the parent, financial contributions, and leisure.

Next we performed a set of χ^2 goodness-of-fit tests for informal care, financial contributions towards formal care, and leisure. For each variable x (time spent helping per family member, proportion of family members offering financial help, and leisure per family member), we simulated x twenty times for each family n and computed the mean \hat{x} and the standard deviation \hat{s} .²⁹ Then we

²⁹We adjusted trivially small predicted values of choice variables (time spent helping per family member, proportion offering financial help, and leisure per family member) to zero.

constructed

$$\chi_{1n}^2 = \frac{(x_n - \hat{x})^2}{\hat{s}^2 + \sigma_m^2} \tag{18}$$

where σ_m^2 is a correction for measurement error. Its construction is discussed in Appendix S3 in Byrne, Goeree, Hiedemann and Stern (2008).³⁰ We then summed χ_{1n}^2 over n . The results of this exercise are presented in Table 14, disaggregated by family size. The χ^2 statistics for time help and leisure are all very large, but the mean residuals are very small. For example, the mean residual on time help for families of size 1 means that we overestimate time help in such families by 0.1 percent on average.

[INSERT TABLE 14 HERE]

The large χ^2 statistics are caused by outliers to a great degree. To reduce the influence of outliers, we censor the χ^2 statistic and adjust the test statistic’s distribution function and critical values appropriately. We present both censored and uncensored results to illustrate the impact of the outliers. Specifically, if we censor each χ_{1n}^2 statistic in equation (18) at the one percent level, i.e.,

$$\chi_{1n}^{2*} = \min \left[\frac{(x_n - \hat{x})^2}{\hat{s}^2 + \sigma_m^2}, 6.63 \right],$$

then the χ^2 statistics reduce to the numbers in the column labeled “Censored.” The next column shows the number of χ_{1n}^{2*} statistics that are actually censored, and the last column turns the censored χ^2 statistic into a standard normal random variable. The results suggest that we are still missing some aspect of decision making with respect to informal care though not in terms of average caregiving time. On the other hand, after controlling for a small number of outliers, we are predicting time help, financial help, and leisure decisions quite accurately.

7.4 Policy Experiments

We consider the effects of six simulated experiments on family behavior given the parameter estimates reported in Table 8. The six experiments involve:

When the sample value and the predicted value of the variable is zero, the chi-square component for a family is a ratio that explodes. We do not include these cases in the test statistic. For financial help, there are many such cases hence the small degrees of freedom for the financial help statistics.

³⁰We added a measurement error term to the goodness-of-fit components to prevent the denominators in any of the ratios from exploding. However, for financial help, the data do not indicate what the magnitude of this measurement error should be. We set it to one, which allows us to include all observations in the goodness-of-fit statistics. Note that we are not including many zero residuals at corners. Exclusion of zero residuals significantly biases our test statistics upward making it more likely we would reject a good fit of the model to the data.

1. providing a subsidy of qF to each parent that must be used for formal care (formal care stamps);
2. providing a subsidy of F to each child or child-in-law for each unit of time she provides informal care;
3. providing a subsidy of F for each dollar spent on formal care (reduction in the price of formal care);
4. providing a lump sum of F to the parent;
5. increasing Ψ , the income limit for Medicaid; and
6. providing a subsidy of qF to each parent for each ADL problem; this subsidy must be used for formal care.

Most real-world experiments are similar to experiment (6)³¹ in that they target people who are likely to need help and provide services that are quite restricted in nature. However, many state programs provide pay for family members who are providing services similar to a formal caregiver,³² making experiment (2) relevant, and, as discussed in Section 3, many states have experimented with changing Medicaid income limits for long-term care such as in experiment (5). As discussed in Benjamin (2001), Germany has a policy similar to experiment (4); several US states are experimenting with the German policy (with some limits on eligibility similar to those in experiment (6)). We have found no examples in the literature providing subsidies similar to experiment (3).

Given the small marginal product of formal and informal care on Q implied by the parameter estimates in Table 8, almost all of the policy experiments would have essentially no effect on behavior. Experiment (1) suggests that formal care stamps would have little impact on expenditures for formal care. Most families without formal care expenditures prior to the experiment would exhaust their formal care stamps but spend no out of pocket funds on formal care. To a large degree, those with formal care expenditures would replace their own expenditures with program expenditures with little effect on the level of formal care or informal care.³³

Experiments (2) and (3) essentially reduce the price of informal and formal care. However, since the family resources expended on both are small and both marginal products are small, the effects of the subsidy would be small. Experiment (4) indicates that a lump sum subsidy to the parent would be used to supplement consumption. This is similar to results discussed in Benjamin (2001). Thus, a lump sum subsidy would have very little effect on formal or informal care or the health (Q) of the parent. Experiments (5) and (6) are small deviations of experiment (1) and would have similar though smaller effects.

³¹See Weissert, Cready, and Pawelak (1988) for a survey of US demonstration projects and Johri, Beland, and Bergman (2003) for a survey of international demonstration projects.

³²See Exhibit 1 in Benjamin (2001).

³³See Pezzin, Kemper, and Reschovsky (1996) for similar results.

Overall, the results of these experiments suggest that variation in state Medicaid policy would have little effect on long-term care decisions. The average policy results are very small because most families are at a corner solution and it would take a very expensive policy to move them away from the corner. For the most part, the proposed policies result in an increase in utility for those who were already providing formal and/or informal care and little change in behavior among everyone else. These results are consistent with results in ES where no significant state effects were found but inconsistent with Cutler and Sheiner (1993) that found small macro effects. We measure the effect of policy changes given that respondents reside in the community and, hence, under some situations, underestimate the effect of changes in policy on community-based care giving. For example, policy changes with regard to Medicaid income limits or subsidies for home health care may imply different choices for community-based care versus institutionalization. Institutional care may be a decision under some policy parameters, while other policy parameters may induce families to care for the elderly parent at home.

8 Conclusions

We develop and estimate a game-theoretic model of families' decisions concerning the provision of informal and formal home health care for elderly individuals. In addition to consumption and leisure, a family member's utility depends on the health quality of elderly relatives as well as her own provision of informal care. Each individual or married couple makes caregiving decisions conditional on the decisions of the other family members. We use the first-order conditions of the model to solve for the errors as relatively simple functions of the parameters and construct the corresponding likelihood function.

The structure of the model allows us to distinguish among several underlying explanations for patterns in care provision. In particular, caregiver burden, caregiving effectiveness, and opportunity costs vary within and across families. Also, the model allows for both self-interest and altruism in the sense that family members value their own consumption and leisure as well as the health quality of their elderly relatives. Our results suggest that most family members, especially children and children-in-law, consider caregiving burdensome, that informal care, especially care provided by a spouse, has a small effect on health quality, and that children care about their parents' health quality. Thus, the tendency of spouses rather than adult children to provide care is attributable to the lower burden experienced by spouses in the caregiving role rather than differences in care effectiveness between spouses and children or selfishness on the part of children. Apparently, the closer bonds between parents and their own children than between parents and children-in-law enhance the quality of care and diminish the burden experienced by members of the younger generation. As expected, children with higher opportunity costs of time exhibit lower caregiving probabilities than their siblings or peers. Interestingly, however, we find that the greater reliance on formal care among more highly educated elderly individuals

is due to differences in care effectiveness rather than income.

The structure of the model sheds light on the roles of gender, marital status, family size, and race. Overall daughters display higher care probabilities than do sons. Our decompositions suggest that differences in opportunity costs, quality, and burden account for much of the gender gap in informal care provision. Unmarried daughters display higher overall care probabilities than their married counterparts, regardless of family size. In relatively small families, this pattern persists even in the absence of differences in opportunity costs, quality, and/or burden. Overall daughters and their husbands are more likely to provide care than are sons and their wives. However, in the absence of quality differences, the opposite pattern would prevail. Racial differences in adult children's opportunity costs and elderly parents' health quality contribute to the greater prevalence of informal care provision in black relative to white families, offsetting differences in quality and burden of care.

Goodness of fit tests show that our model fits the data fairly well. In addition, we fail to reject the hypothesis that there is no additional variation across states not captured in the model. This result suggests that our simplification of the Medicaid benefit structure performs well.

The structural nature of the estimates enables us to perform policy experiments similar to those proposed by policymakers. For example, we simulate the provision of a lump sum that can be spent only on care as well as price subsidies for informal and formal home health care. As a result of the relative ineffectiveness of both forms of care, we find little effect of these policy changes.

Although our results yield important insights concerning families' elder care decisions, several limitations are worth noting. First, the 1993 wave of AHEAD data does not include any nursing home residents. The inclusion of nursing home residents in subsequent waves will enable us to investigate the effects of proposed or actual policies on the use of institutional care. For example, subsidies for home health care may induce some families to care for the elderly at home rather than in an institution. Second, the survey instrument used in subsequent waves elicits more information about caregivers, enabling us to further improve our model. Third, our use of a static model obscures the dynamic aspects of families' care decisions. The availability of panel data will enable us to estimate a dynamic extension of our structural model. Using several waves of AHEAD data, future work will explore whether family members specialize according to their comparative advantages in market production, caregiving, or other forms of nonmarket production or whether siblings take turns providing informal care. If children take turns caregiving, the use of panel data will enable us to examine possible causes of this behavior including burnout. Finally, our current work does not adequately address differences by race. In future work, we plan to estimate separate models by race to examine the extent to which characteristics such as child gender operate differently by race.

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Appendix

A Construction of Child Caring Time

A key issue in estimation concerns the interpretation of data on caregiving time t_{jik} . In the survey, there are two relevant questions:

- 1) How many days per week does the helper provide help?; and
- 2) How many hours per day does the helper provide help on days when she helps?

While the responses to the second question provide a continuous measure of hours per day, responses to the first question are categorical: a) every day, b) several times a week, c) once per week, d) less than once per week, and e) never. We can use the answers to these two questions to construct a “pseudo” continuous variable:

$$t_{jik} = \begin{cases} 7\pi_{jik}/168 & \text{if she helps every day} \\ 3.5\pi_{jik}/168 & \text{if she helps several times a week} \\ \pi_{jik}/168 & \text{if she helps once per week} \\ 0.5\pi_{jik}/168 & \text{if she helps less than once per week} \\ 0 & \text{if she never helps} \end{cases} \quad (19)$$

where π_{jik} is the answer to the second question.³⁴

Unfortunately, the AHEAD respondents were asked about help from children only if they had an ADL or IADL problem. This feature of the survey design may bias the amount of reported care downwards. However, it is reasonable to assume that parents needing care from children are likely to have an ADL or IADL, and, at the time we constructed our data, there were no better data available.

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³⁴Alternatively, we could have set up bracketed amounts that are truer to the nature of the first question. Using the brackets is much harder, and it adds precision only for two out of the five categories. These two occur for 949 helpers out of a total of 3144 helpers (30.1 percent). Stern (1994) shows that the added precision gained by treating the variable as bracketed is very small in a similar problem.

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Tables and Figures

Table 1 Dropped Households	
Selection Criteria	Number of Observations
Total Households	6047
More than Five Children	625
Missing Child Variable	1008
Missing Parent Variable	108
Working Respondent	270
Respondents Helping Each Other	25
Small Minority Groups	350
Coding Errors	78
Sample Size	3583

Table 2 Selected Characteristics of Respondents		
Characteristic	Male	Female
Age	76.73	76.30
Education	11.73	11.80
Black	0.07	0.10
Living Children	2.06	2.02
Married	0.72	0.42
Number of ADL problems	0.52	0.71
At Least 1 ADL problem	0.23	0.31
Number of IADL problems	0.36	0.34

Table 3 Child Characteristics of Respondents	
Characteristic	Mean
Age	47.01
Male	0.490
Education	13.98
Married	0.698
Number of Children	1.985
Live with Parent	0.06
Live More Than 10 Miles from Parent	0.62
Imputed Weekly Wage	\$452

Note: We also observe bracketed time spent helping respondents and labor force participation of the child and spouse of the child.

Table 4 Ln Wage Estimates					
Variable	Estimate		Variable	Estimate	
Constant	0.028 (0.072)		Male	0.099 (0.030)	**
Years of Schooling if Not Completed High School	0.035 (0.006)	**	Married	0.028 (0.029)	
High School Diploma	0.540 (0.052)	**	White	0.066 (0.022)	**
Some College	0.680 (0.053)	**	Male*Married	0.090 (0.042)	**
College Degree	0.978 (0.053)	**	Male*White	0.022 (0.033)	
> College Degree	1.086 (0.054)	**	Married*White	-0.035 (0.032)	
Age	0.066 (0.002)	**	Male*Married*White	0.093 (0.045)	**
Age ²	-0.001 (0.000)	**			
$R^2 = 0.34$					

Notes:

1. The dependent variable is ln wage.
2. The numbers in parentheses are standard errors.
3. Double starred items are significant at the 5 percent level.
4. The education variables refer to highest education level attained. The first variable is a slope conditional on not finishing high school, and the others are dummy variables.

Table 5 Characteristics of Care Provision for Families of Various Sizes								
Type of Family	No Children		Number of Children ¹					Total
	Single	Married	1	2	3	4	5	
Percentage of All Families	17.8	3.7	20.8	27.8	16.9	9.0	3.9	100
Percent of Families:								
Receiving Care	5.6	38.1	26.3	24.7	25.7	26.1	22.9	22.3
Receiving Formal Care ²	100	9.8	21.8	12.9	12.2	8.2	3.1	17.8
Receiving Informal Care ²		98.0	88.3	93.5	96.8	100	100	89.9
Receiving Formal and Informal Care ²		7.8	10.2	6.5	9.0	8.2	3.1	7.7
Percent of Families Where:								
Children Help Pay for Care ³			11.6	12.5	5.3	0	0	8.7
Spouse Provides Informal Care ⁴		100	48.9	62.9	63.6	63.5	68.8	62.6
Children Provide Informal Care ⁴			54.0	40.1	43.7	42.4	40.6	41.7
Multiple Children Provide Informal Care ⁵				9.7	16.7	19.4	23.1	14.4
Children and Spouse Provide Informal Care ⁴			2.9	3.0	7.3	5.9	9.4	4.6
Average Hours Per Week:								
Informal Care Provided by Spouse ⁶		26.8	25.8	25.8	24.3	27.1	34.4	26.2
Informal Care Provided by Children ⁷			21.3	23.7	27.5	21.9	16.8	23.5

Notes:

1. Includes families with single and married respondents.
2. As share of families with respondents receiving any care.
3. As share of families with respondents receiving formal care.
4. As share of families with respondents receiving informal care.
5. As share of families with children providing informal care.
6. Average over families with spouse providing informal care.
7. Average over families with children providing informal care.

Table 6			
Informal Care Provision Probit Coefficient Estimates			
Explanatory Variable	Estimate		Std. Err
Constant	-3.478	**	0.466
Parent Characteristics			
Age	0.274	**	0.007
Father	-0.310	**	0.093
White	0.072		0.106
Married	-0.606	**	0.085
Education	-0.047	**	0.011
Number of ADL Problems	0.230	**	0.018
Child Characteristics			
Age	0.002		0.005
Male	0.033		0.068
Married	0.130		0.081
Education	-0.012		0.015
Number of Children	-0.014		0.023
Oldest Child	0.073		0.073
Wage	-0.000		0.000
Log Likelihood	-801.5		
Number of Observations	7,562		

Notes:

1. Dependent variable is equal to one if child or child's spouse (if applicable) helped the parent.
2. Double starred items are significant at the 5 percent level.

Table 7					
Estimates of Model With No Covariates					
Variable	Estimate		Variable	Estimate	
$\log \alpha$	-7.912 (0.415)	**	β_{4i}	-0.452 (0.072)	**
$\log \mu$	-4.547 (0.451)	**	γ	-11.740 (3.049)	**
$\log Z$	-4.674 (0.068)	**	$\log \sigma_{\eta X}$	-0.182 (0.159)	
$\log(-\beta_0)$	9.031 (0.709)	**	$\log \sigma_{\eta L}$	0.215 (0.008)	**
$\log \beta_{10}$	-3.216 (3.044)		$\log \sigma_{\eta t}$	0.105 (0.020)	**
$\log \beta_2$	0.000 Restricted		$\log \sigma_u$	8.864 (0.796)	**
β_{40}	-0.114 (0.047)	**	ρ_L	0.900 Restricted ²	
$\log \beta_{1i}$	4.788 (0.427)	**	ρ_t	-0.058 0.056 ²	
$\log \beta_{3i}$	0.657 (0.050)	**			

Notes:

1. Numbers in parentheses are standard errors. Single starred items are significant at the 10 percent level, and double starred items are significant at the 5 percent level.
2. ρ_L and ρ_t are set equal to

$$\rho_r = 1.8 \frac{\exp\{\lambda_r\}}{1 + \exp\{\lambda_r\}} - 0.9$$

for $r = L, t$ to insure nice properties of the model. The value of λ_L is restricted to 10. The estimate of λ_t is -0.128 (with a standard error of 0.125) which implies that the standard error of ρ_t is 0.059.

3. The log likelihood value is -11195.373 .

Table 8									
Estimates with Covariation in $\log \alpha$ and β_4									
Variable	Estimate on $\log \alpha$		Estimate on $\log Z$		Estimate on β_4		Estimate on $\log \beta_1$		Estimate on $\log \beta_3$
Parent Characteristics									
Constant	-3.704 (2.156)	*	0.662 (0.838)		-3.530 (0.181)	**	0.616 (0.415)		
Age/100	11.506 (0.422)	**	6.814 (0.170)	**	4.311 (0.157)	**			
Education	-0.012 (0.006)	**							
White	0.118 (0.124)		0.114 (0.113)		0.027 (0.134)				
Married			0.482 (0.082)	**					
# ADL Problems	-0.023 (0.140)		-0.184 (0.143)		-0.172 (0.041)	**			
Mother	-1.438 (0.622)	**	-1.630 (0.614)	**	0.584 (0.064)	**			
Spouse Characteristics									
Age/100	-29.532 (1646.66)								
# ADL Problems	0.330 (0.341)								
Child Characteristics									
Constant	0.949 (2.295)				-5.073 (0.593)	**	1.364 (0.417)	**	0.058 (0.065)
Age/100	-3.797 (0.153)	**			6.159 (0.509)	**			
Male	-0.323 (0.377)				-0.269 (0.385)				
Biological ²	0.331 (0.229)				0.431 (0.339)				
Biological *Male ²	0.311 (0.394)				-0.042 (0.432)				
Education	0.009 (0.017)				-0.003 (0.030)				
Married	-0.154 (0.096)	*			0.050 (0.163)				
# Kids	-0.025 (0.032)				0.022 (0.055)				
Oldest	0.218 (0.084)	**			-0.245 (0.140)	*			

Table 8 (continued)					
Estimates with Covariation in $\log \alpha$ and β_4					
Variable	Estimate		Variable	Estimate	
$\log \mu$	-3.576 (0.285)	**	$\log \sigma_{\eta t}$	-0.014 (0.020)	
γ	-0.853 (0.067)	**	$\log \sigma_u$	10.159 (0.929)	**
$\log \beta_0$	10.335 (0.814)	**	ρ_L	0.900 Restricted ³	
$\log \sigma_{\eta X}$	0.135 (0.041)	**	ρ_t	0.622 0.066 ³	**
$\log \sigma_{\eta L}$	0.246 (0.009)	**			

Notes:

1. The numbers in parentheses are standard errors. Single starred items are significant at the 10 percent level, and double starred items are significant at the 5 percent level.
2. Technically the term biological is not correct since these include adopted children. We use this term to distinguish between children and children-in-law.
3. ρ_L and ρ_t are set equal to

$$\rho_r = 1.8 \frac{\exp \{\lambda_r\}}{1 + \exp \{\lambda_r\}} - 0.9$$

for $r = L, t$ to insure nice properties of the model. The value of λ_L is restricted to 7. The estimate of λ_t is 1.670 (with a standard error of 0.280) which implies that the standard error of ρ_t is 0.066 .

4. The log likelihood value is -11357.01 .

Table 9 Moments of Behavior				
	Parent or Spouse		Children	
Variable	Mean	Std. Dev	Mean	Std. Dev
$\log \alpha$	-17.814	1.966	3.403	1.044
$\log \beta_1$	0.616	0.000	1.364	0.000
$\log \beta_3$	0.000	0.000	0.058	0.000
β_4	-0.061	0.366	-2.194	0.544
Utility	23.590	19.952	23.970	18.065
\log Health Quality	5.102	0.984		
$\partial(\log \text{Health Quality})/\partial(\text{Informal Care})$	0.000	0.000	0.206	0.129
$\partial(\log \text{Health Quality})/\partial(\text{Formal Care})$	0.000	0.000		

Note: The measure of formal care includes care financed by the child or parent. The model does not allow for the impact of formal care to depend upon who paid for it.

Table 10 Predicted Characteristics of Care Provision for Families of Various Sizes								
Type of Family	No Children		Number of Children ¹					Total
	Single	Married	1	2	3	4	5	
Percentage of All Families	17.8	3.7	20.8	27.9	16.9	9.1	3.9	100
Percent of Families:								
Receiving Care	0.0	33.6	16.0	18.4	20.4	22.1	20.9	16.0
Receiving Formal Care ²	100	0.3	0.2	0.3	0.3	0.3	0.3	0.3
Receiving Informal Care ²		99.8	99.9	99.8	99.8	99.8	99.7	99.8
Receiving Formal and Informal Care ²		0.1	0.0	0.1	0.1	0.1	0.0	0.1
Percent of Families Where:								
Children Help Pay for Care ³			25.0	10.0	37.5	0.0	0.0	14.7
Spouse Provides Informal Care ⁴		100	61.9	67.8	63.6	64.7	61.3	67.4
Children Provide Informal Care ⁴			39.0	34.0	39.3	37.7	43.6	34.5
Multiple Children Provide Informal Care ⁵				13.1	17.7	21.7	16.9	12.6
Children and Spouse Provide Informal Care ⁴			1.9	2.9	4.9	5.1	7.0	3.4
Average Hours Per Week:								
Informal Care Provided by Spouse ⁶		91.7	63.1	68.8	64.3	66.8	63.9	67.9
Informal Care Provided by Children ⁷			21.2	26.0	27.6	30.3	29.0	26.1

Notes:

1. Includes families with single and married respondents.
2. As share of families with respondents receiving any care.
3. As share of families with respondents receiving formal care.
4. As share of families with respondents receiving informal care.
5. As share of families with children providing informal care.
6. Average over families with spouse providing informal care.
7. Average over families with children providing informal care.

	# Obs	All Effects	Just Wage Effect	Just Quality of Care Effect	Just Burden Effect	No Effects
<u>One Child Families</u>						
Single Daughters	165	-3.864	-3.864	-3.864	-3.864	-3.864
Single Sons	110	-5.735	-4.079	-4.960	-4.454	-3.951
Married Daughters	238	-4.920	-4.523	-4.519	-4.656	-4.352
Married Sons	238	-4.947	-3.763	-4.133	-3.928	-3.454
<u>Two Children Families</u>						
Single Daughters	361	-4.737	-4.737	-4.737	-4.737	-4.737
Single Sons	238	-6.810	-5.181	-5.941	-5.636	-5.036
Married Daughters	675	-5.430	-4.997	-5.008	-5.163	-4.827
Married Sons	732	-6.103	-4.722	-5.137	-4.996	-4.372
<u>Three Children Families</u>						
Single Daughters	282	-5.071	-5.071	-5.071	-5.071	-5.071
Single Sons	226	-6.394	-4.767	-5.534	-5.263	-4.645
Married Daughters	631	-5.744	-5.272	-5.378	-5.446	-5.088
Married Sons	686	-6.427	-5.033	-5.438	-5.316	-4.655
<u>Four Children Families</u>						
Single Daughters	205	-5.582	-5.582	-5.582	-5.582	-5.582
Single Sons	210	-7.210	-5.366	-6.221	-5.968	-5.237
Married Daughters	457	-5.979	-5.500	-5.519	-5.691	-5.324
Married Sons	432	-6.822	-5.367	-5.712	-5.628	-4.936
<u>Five Children Families</u>						
Single Daughters	99	-5.922	-5.922	-5.922	-5.922	-5.922
Single Sons	93	-6.474	-4.711	-5.553	-5.264	-4.591
Married Daughters	247	-6.016	-5.548	-5.597	-5.772	-5.405
Married Sons	261	-6.880	-5.371	-5.729	-5.624	-4.917

Notes:

1. Each element in the table is the $\log \Pr [t > 0 | Gender, Effect]$.
2. The elements corresponding to single children use the $\log \Pr$ [that child provides care], and the elements corresponding to married children use the $\log \Pr$ [that child or the spouse of that child provides care].

	All Effects	Just Wage Effect	Just Quality of Care Effect	Just Burden Effect
Single Children				
One Child Families	-1.784	-0.128	-1.009	-0.503
Two Children Families	-1.774	-0.145	-0.905	-0.600
Three Children Families	-1.749	-0.122	-0.889	-0.618
Four Children Families	-1.973	-0.129	-0.984	-0.731
Five Children Families	-1.883	-0.120	-0.962	-0.673
Married Children				
One Child Families	-0.925	-0.138	-0.512	-0.170
Two Children Families	-1.128	-0.180	-0.584	-0.288
Three Children Families	-1.116	-0.194	-0.493	-0.303
Four Children Families	-1.231	-0.255	-0.581	-0.325
Five Children Families	-1.352	-0.311	-0.620	-0.340

Notes:

1. Each element in the table is the

$$\begin{aligned}
 & (\log \Pr [t > 0 \mid \text{Male}, \text{Effect}] - \log \Pr [t > 0 \mid \text{Female}, \text{Effect}]) \\
 & - (\log \Pr [t > 0 \mid \text{Male}, \text{No Effects}] - \log \Pr [t > 0 \mid \text{Female}, \text{No Effects}]).
 \end{aligned}$$

These can be turned into percentage changes by exponentiating and subtracting one.

2. The elements corresponding to single children use the $\log \Pr$ [that child provides care], and the elements corresponding to married children use the $\log \Pr$ [that child or the spouse of that child provides care].

Table 13				
Decomposition of Child Race Effects on $\frac{\Delta^2 \log \Pr[t>0]}{\Delta \text{Effect} \Delta \text{White}}$				
	All Effects	Just Wage Effect	Just Quality of Care Effect	Just Burden Effect
Single Children				
One Child Families	0.290	-0.067	0.315	0.042
Two Children Families	0.257	-0.071	0.285	0.042
Three Children Families	0.253	-0.071	0.279	0.045
Four Children Families	0.278	-0.077	0.303	0.051
Five Children Families	0.254	-0.083	0.292	0.045
Married Children				
One Child Families	0.185	-0.104	0.243	0.046
Two Children Families	0.195	-0.102	0.250	0.048
Three Children Families	0.192	-0.112	0.253	0.051
Four Children Families	0.215	-0.088	0.250	0.051
Five Children Families	0.230	-0.098	0.274	0.054

Notes:

1. Each element in the table represents

$$\begin{aligned}
 & (\log \Pr [t > 0 \mid \textit{White}, \textit{Effect}] - \log \Pr [t > 0 \mid \textit{Black}, \textit{Effect}]) \\
 & - (\log \Pr [t > 0 \mid \textit{White}, \textit{No Effects}] - \log \Pr [t > 0 \mid \textit{Black}, \textit{No Effects}]).
 \end{aligned}$$

2. The elements corresponding to single children use the $\log \Pr$ [that child provides care], and the elements corresponding to married children use the $\log \Pr$ [that child or the spouse of that child provides care].

Table 14						
χ^2 Goodness of Fit Tests						
Family Size	df	Mean Residual	χ^2 Statistic	Censored	# Censored Obs	Normalization
Time Help						
1	132	-0.09	50.65	50.65	0	-5.01
2	750	-0.02	229.56	229.56	0	-13.44
3	1003	-0.02	333.08	333.08	0	-14.96
4	608	0.00	1664.01	206.81	2	-11.50
Financial Help						
1	3	-0.15	0.16	0.16	0	-1.16
2	7	0.54	4084151.70	6.74	1	-0.07
3	17	-0.03	0.37	0.37	0	-2.85
4	18	-0.04	0.31	0.31	0	-2.95
Leisure						
2	750	0.04	216.10	216.10	0	-13.79
3	1003	0.13	981.14	981.14	0	-0.49
4	608	0.17	1256.75	1249.97	4	18.41

Notes:

1. A family of size M has $M - 1$ children.
2. The statistics reported in the column labeled "Normalization" are normalized by subtracting off the mean of the censored χ_{df}^2 , $0.978 \cdot df$, and dividing by the standard deviation, $\sqrt{1.722df}$. The relevant general formula is

$$\begin{aligned}
 E\chi_{1c}^2 &= F_3(c) + c[1 - F_1(c)]; \\
 E(\chi_{1c}^2)^2 &= 3F_5(c) + c^2[1 - F_1(c)]
 \end{aligned}$$

where χ_{1c}^2 is a χ^2 random variable with one degree of freedom censored at c and $F_{df}(c)$ is the χ^2 distribution function with df degrees of freedom evaluated at c .

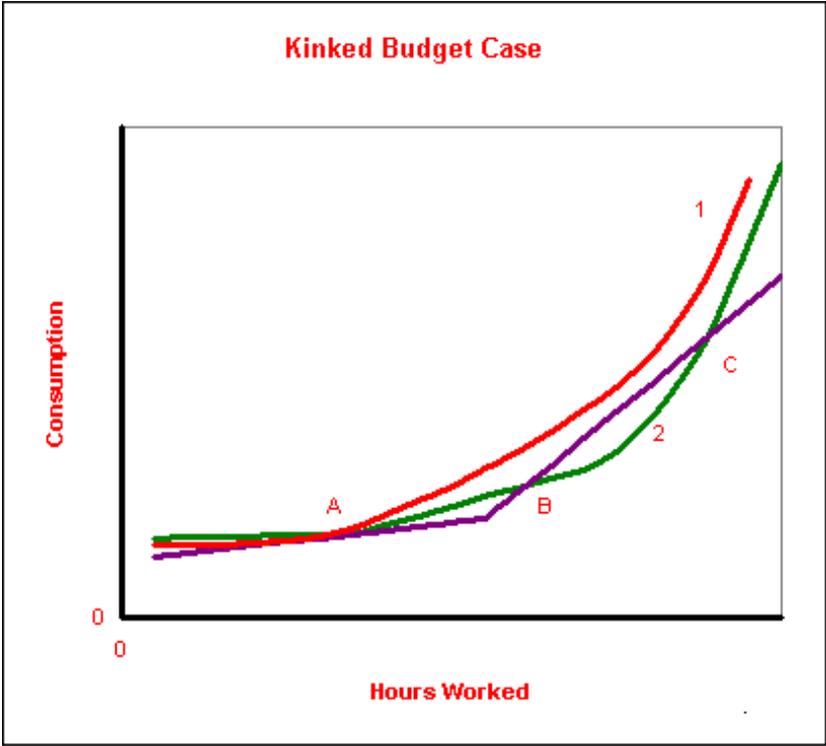


Figure 1